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Value of Real-Time Information on Traffic Congestion

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Abstract

In this paper, we derive the improvement in benefit by using real time information on traffic conditions on each road. By using real time information, an individual can choose the less congested path and thus get to the destination faster than otherwise. When we have real time information, a route which takes more time on the average can sometimes be used for an individual's benefit to reduce the congestion cost. In addition to an individual's benefit improvement, real time information improves the utilization of each road in a traffic system. By inducing more balanced traffic flow on each road, we can reduce the whole society's congestion cost. Although this paper analyzes traffic systems which have multiple routes to the destination, the methodology can be corresponded to a dynamic routing problem in a queueing system with multiple queues and servers.

1. Introduction

When a person drives a car from a starting point to a destination, he may choose one of many routes to the destination. If he happens to be caught in a traffic jam in the path he chose, he wishes to have chosen another path which is less congested. If he had chosen the less congested path

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from the beginning, he might have saved lots of time and cost. This is possible if he can get the real-time information on the traffic conditions of many possible routes to the destination. This kind of situation can occur to a truck company which transports several products to many customers. When JIT delivery is required, choosing a right path will save lots of cost. We first consider the case where the departure time is given and a customer tries to analyze the benefit of real-time information in minimizing the transportation cost(section 2). And then we deal with the case where the departure time is also a decision variable in minimizing total transpotation cost(section 3).

In this paper we study the value of real-time information on traffic conditions. If we get the real-time information on the traffic congestion of many roads, we may choose the least congested path and thus can arrive at the destination faster and in a less costly way. We consider the value of information in terms of saving the mean(first moment) time used to arrive at the destination. We discuss the implications of this paper for both policy makers and individual customers. The other areas to which our analysis can be applied will also be discussed in the concluding remarks.

The incompatibility between individual optimization and social optimization in a queueing system was studies in Naor[1969]. He showed that without any extra cost more individuals would enter the queueing system for service than socially optimal level. This is beacuse by entering the queueing system an individual incurs externality cost to all the other users of the queueing system but he takes into account only the cost imposed on himself, which is merely a fraction of the total externality cost. In order to achieve the socially optimal arrival rate of customers, Naor suggested levying tolls whose value plus the individual cost(the effective cost) woluld induce the socially optimal level. Another way to reduce the discrepancy between the individual optimization and the social optimization was suggested in Nam [1995]. He suggested that giving imperfect information on the queue states would induce fewer entrance to the queueing system and thus alleviate the discrepancy. Mendelson[1985] studied pricing mechanisms for another queueing system which offers computer service. Incorporating queueing effects to the microeconomics of computing center management, he dealt with pricing schemes under various settings. In terms of the subject covered, Yoshino et al.[1995] is closely related to our model. They gave a real case showing benefit improvement by using real time information on the traffic conditions of a highway.

In our paper, we consider a queueing system(specifically a traffic system, for example) where there are more than one servers. Each server can be corresponded to traffic route to a destination. Previous researches focused on attaining socially optimal level of congestion by using pricing mechanism (Naor[1969], Mendelson[1985]) or by giving more vague information on queue states(Nam[1995]). In the current paper, we specifically deal with a traffic system and derive the benefit improvement of an individual when she / he gets the real-time information on traffic conditions and thus chooses the less congested path. Our model newly considers a queueing system (specifically traffic system) with multiple servers(paths). In the previous researches, they were concerned whether a customer enters the service system or not. In our model, we assert when we get the real time information on the queue states(number of cars in each path), we would be better off by avoiding more congested path and arriving at the destination faster than otherwise. In addition to this individual benefit improvement, the real time information on traffic conditions would induce more effective use of traffic routes by balancing usage across several paths. This will be called social benefit improvement. Although we specifically deal with a traffic system, we can extend the idea to the scheduling problem for multi-machine job shop.

2. Case Where Departure Time Is Fixed

2.1 Individual Benefit Improvement

We use the mean time spent for arriving at the destination as the criterion for judging which path is better. Obviously we also can interpret the mean time as mean cost for arriving at the destination. Let us assume that there are two routes to a destination for simplicity. And the time it takes to get to destination through the first path is denoted as a random variable X, and the time through the other path as Y. We can generalize this interpretation such that X denotes the cost felt by a customer due to congestion. Readers should note that *time* in this paper can also mean congestion cost. It is also assumed that X and Y are independent. The number of paths(currently two) can be enlarged without any difficulty. The notation $\wedge$ menas taking the minimum between the two(i.e., $x \wedge y \equiv \min\{x, y\}$).

Lemma 1

$$E(X \wedge Y) \leq E(X) \wedge E(Y).$$

Proof: Since $X \wedge Y \overset{a.s.}{\leq} X$ and $X \wedge Y \overset{a.s.}{\leq} Y$ (*a.s.* means almost surely or with probability 1), the result follows. □

More generally, since $X \wedge Y \overset{a.s.}{\leq} X$ implies that $P(X \wedge Y \leq a) \geq P(X \leq a)$ for all a, we know X first-order stochastically dominates $X \wedge Y$. When a person does not have any information on the traffic conditions of paths 1 and 2, he will choose either of them randomly or one which is faster in the mean. But if he is informed on the current traffic conditions on both of the roads, he will choose the path which is less congested and thus can arrive at the destination faster. In this case the time required to arrive at the destination will be $X \wedge Y$. We can define the value of real-time information on the traffic conditions in this case as $V_2 = \alpha E(X) + (1-\alpha)E(Y) - E(X \wedge Y)$, where α is the chance of customer's using path 1. When we have several(say n) paths which lead to the destination, the value of information becomes even larger: $V_n = \sum_{i=1}^{n} \alpha_i E(X_i) - E(X_1 \wedge X_2 \cdots \wedge X_n)$, where α_i is the chance of customer's using path i. By Lemma 1, we know $V_n > 0$.

Example 1: Let X_i denote the time or cost for using path i from the starting point to the destination($i=1, 2, \ldots, n$). All the times required are assumed to follow the same exponential distribution. That is, $X_i \sim Exp(\lambda)$. Without any information on the traffic states of n routes the expected time for travel is $1/\lambda$. When we get the real-time information on the traffic states of each road, we can choose the one with least congestion and thus the time it takes is

$$\min_{i=1,2,\ldots,n} \{X_i\}$$

We know that

$$\min_{i=1,2,\ldots,n} \{X_i\} \sim Exp(n\lambda).$$

In this case the mean traveling time is $1/(n\lambda)$ and thus reduced by n times. The value of real-time information becomes $\lambda(1-1/n) > 0$.

We have seen that we can in this case improve the mean time spent for arriving at the destination by n(the number of possible routes) times. Thus if we can use the real-time information on the traffic conditions, we can cover n times more customers by using the same number of vehicles. Or we can serve the same number of customers with only $1/n$ number of vehicles and drivers. In many cases there exist many possible routes to the destination and you can see that improvement can be amazingly large.

2.2 Implication of Using Real-Time Traffic Information

The most important of the result in this section is as follows. Even though some paths are inferior in terms of mean time required for arriving at the destination and thus are excluded by a driver, they can help reduce the time spent for arriving at the destination when the driver can get the real-time information on the traffic conditions of the roads. For example, suppose that there are two paths, paths 1 and 2, which are available to you. You usually take path 1 since it takes less time in the mean than path 2, that is $E(X)<E(Y)$ where X and Y represent the random time required for traveling path 1 and path 2 respectively. Sometimes traffic accidents or emergency road works occur on path 1. Without any information on the traffic conditions on paths 1 and 2, you will use path 1 and may spend lots of time caught in a traffic jam. If you have got the real-time information on the traffic congestion on path 1, you could choose path 2 and thus save a lot of opportunity cost even though it would take a little more time than it usually takes(since path 2 is like a detour). For the exponentially distributed travel time, the following example shows the point.

Example 2: Let $X_1 \sim Exp(\lambda_1)$ and $X_2 \sim Exp(\lambda_2)$ and $\lambda_1 > \lambda_2$. Then $E(X_1) = 1/\lambda_1 < E(X_2) = 1/\lambda_2$, and thus path 1 is superior to path 2 in terms of mean value. But when we use the real-time information on the traffic states on paths 1 and 2, the time spent is $\min\{X_1, X_2\}$ and the mean time is $1/(\lambda_1+\lambda_2)$. It is clear that this value is smaller than both $E(X_1)$ and $E(X_2)$. Thus the inferior path in terms of mean(path 2, in this example) can sometimes be chosen for customer's benefit by using the real-time information. The benefit of using real-time information in this case is

$$\alpha E(X_1)+(1-\alpha)E(X_2)-E(\min\{X_1, X_2\})$$
$$=\frac{\alpha}{\lambda_1}+\frac{1-\alpha}{\lambda_2}-\frac{1}{\lambda_1+\lambda_2}>\frac{1}{\lambda}-\frac{1}{\lambda_1+\lambda_2}>0.$$

When there are more than just two paths(in most cases it is true), the benefit of using real-time information on trafic states becomes larger.

2.3 Social Benefit Improvement

In this subsection we consider the social benefit improvement from using real time information on the traffic states. When a customer chooses the faster route using the real time information, he can not only save his own cost but also reduce the social cost as a whole. This reduction in social cost comes from the phenomenon that by not choosing the already congested road but taking the less congested path instead, a customer can avoid escalating the congestion cost in the society as a whole. Thus the real time information makes it possible for customers to choose a path for their own benefit. And this leads the society to use road capacity more effectively.

Till now we only considered the time required for arriving at the destination. We now extend the concept and introduce the cost function. We introduce the inconvenience or congestion cost functions V_1 and V_2. The congestion cost function V_i is a mapping from N_0 to R^+, where $N_0=\{0, 1, 2, \cdots\}$ and $R^+=[0, \infty)$. We assume that $zV_i(z)$ is convex in z, which is generally true for congestion cost functions(if $V_i(z)$ is convex and increasing, then $zV_i(z)$ is clearly convex). For the case where we can approximate V_i such that the domain of V_i is also R^+, we can derive more compact form of conditions corresponding to Lemma 2 and Lemma 3. These corresponding lemmas and various examples are in the appendix. For example, $V_1(x)$ is the congestion cost felt by a customer who observes x cars in path 1 and decides to enter path 1.

The pair of congestion cost functions(V_1, V_2) is called *socially beneficial* if the following conditions are satisfied: For any (x, y) such that $V_1(x)>V_2(y)$,

$$(x+1)V_1(x+1)+yV_2(y)>(y+1)V_2(y+1)+xV_1(x)$$

and for any (x, y) such that $V_1(x)<V_2(y)$,

$$(x+1)V_1(x+1)+yV_2(y)<(y+1)V_2(y+1)+xV_1(x)$$

The conditions above say that by choosing less congested path we save the social cost as a whole. We now give the necessary and sufficient conditions for (V_1, V_2) to be socially beneficial congestion cost functions. For the simplicity of our exposition, we assume that for any x, there exists y such that $V_1(x)=V_2(y)$.

Lemma 2 *Assume that $zV_i(z)$ is convex in z for $i=1, 2$. The necessary and sufficient conditions for (V_1, V_2) to be socially beneficial are: for any (x, y) such that $V_1(x)=V_1(y)$,*

$$(x+1)V_1(x+1)+yV_2(y)=xV_1(x)+(y+1)V_2(y+1).$$

Proof: Define $\triangle_1(x+1)=(x+1)V_1(x+1)-xV_1(x)$, $\triangle_2(y+1)=(y+1)V_2(y+1)-yV_2(y)$, and $k(x, y)=\triangle_1(x+1)-\triangle_2(y+1)$. And let $D^>=\{(x, y): V_1(x)>V_2(y)\}$ and likewise for $D^<$. Then the definition of social beneficiality is equivalent to

$$\text{For any } (x, y) \in D^>, k(x, y) > 0,$$
$$\text{For any } (x, y) \in D^<, k(x, y) > 0.$$

Denoting $E^+=\{(x, y): k(x, y)>0\}$ and $E^-=\{(x, y): k(x, y)<0\}$, two conditions above are $D^> \subset E^+$ and $D^< \subset E^-$. From the convexity of $zV_i(z)$, we know that k is increasing in x and decreasing in y. Using this property, we get the condition that the boundary of $(D^>, D^<)$ and that of (E^+, E^-) should be equal. And thus the conditions can be represented by for any (x, y) such that $V_1(x)=V_2(y)$, $(x+1)V_1(x+1)+yV_2(y)=xV_1(x)+(y+1)V_2(y+1)$.□

2.4 Socially Optimal Traffic Flow

We have considered an individual customer as a decision maker for choosing a path. He chooses a path for his own benefit. For the congestion cost functions which are socially beneficial, his decision on the path for his own benefit is congruent with social benefit improvement.

Now we consider a social agency which should minimize social cost due to congestion. The choices made by each individual customer may not necessarily produce socially optimal solution. Suppose that there are m customers with real time information on traffic conditions and they choose one of paths 1 and 2. The agency wants x^* customers to choose path 1 and the other $m-x^*$ to choose path 2, where x^* satisfies the following condition:

$$x^*=\arg\min_x \{xV_1(x)+(m-x)V_2(m-x)\}.$$

Denoting the total congestion cost in the society as $h(x)=xV_1(x)+(m-x)V_2(m-x)$ and $\triangle h(x)=h(x)-h(x-1)$, we give the optimality conditions for the above cost minimization in the following lemma:

Lemma 3 *Assume that $zV_i(z)$ is convex in z for $i=1, 2$. When there are m customers in the social system, and V_1 and V_2 are congestion cost functions for paths 1 and 2 respectively, the condition for x^* to be the unique socially optimal flow amount for path 1 is as follows:*

$$\triangle h(x^*)<0 \text{ and } \triangle h(x^*+1)>0.$$

Proof: Since $xV_1(x)$ and $yV_2(y)$ are assumed to be convex, $h(x)$ is also convex. Therefore the first order necessary condition is also sufficient for the minimization. For x^* to be the unique solution for minimizing $h(x)$, the first order condition is $\triangle h(x^*)<0$ and $\triangle h(x^*+1)>0$. □

Suppose that there are *sufficient* number of customers with real time information on traffic states. Here by *sufficient*, we mean that there are enough number of customers with real-time information trying to reduce their own costs and they can induce the equilibrium of balanced congestion costs along two paths. By trying to save their own cost, they will choose the less congested path. Thus if there are sufficient number of customers with real-time information, the traffic flow amount choosing each path will satisfy $V_1(\tilde{x})=V_2(\tilde{y})$ at equilibrium, where $\tilde{x}+\tilde{y}$ is the total number of customers using paths 1 and 2. That is, the congestion cost for each path should be the same at equilibrium. We now can ask a question whether the equilibrium of $(\tilde{x}, \tilde{y})$ is socially optimal. The choice for the path made by an individual for his own benefit does not necessarily give a socially optimal solution. The next theorem states that for a socially beneficial congestion cost function pair, the equilibrium induces the socially optimal solution as well.

Propostion 1 *Assume that $zV_i(z)$ is convex in z for $i=1, 2$. When (V_1, V_2) is socially beneficial and there are sufficient number of customers with real time information who are going to use one of paths 1 and 2, the equilibrium state reached by each individual's choice is socially optimal.*

Proof: Since each individual would choose less congested path between the two, the amount of customers on each path, $(\tilde{x}, \tilde{y})$, should satisfy $V_1(\tilde{x})=V_2(\tilde{y})$ and $\tilde{x}+\tilde{y}=m$(the total number of customers). Since (V_1, V_2) is assumed to be socially beneficial, we get by Lemma 2

$$(\tilde{x}+1)V_1(\tilde{x}+1)-\tilde{x}V_1(\tilde{x})=(\tilde{y}+1)V_2(\tilde{y}+1)-\tilde{y}V_2(\tilde{y}).(*)$$

It now suffices to show that

$$\tilde{x}V_1(\tilde{x})+\tilde{y}V_2(\tilde{y})<(\tilde{x}-1)V_1(\tilde{x}-1)+(\tilde{y}+1)V_2(\tilde{y}+1).$$
$$\tilde{x}V_1(\tilde{x})+\tilde{y}V_2(\tilde{y})<(\tilde{x}+1)V_1(\tilde{x}+1)+(\tilde{y}-1)V_2(\tilde{y}-1).$$

From (*), these two conditions are equivalent to

$$2\tilde{x}V_1(\tilde{x})<(\tilde{x}-1)V_1(\tilde{x}-1)+(\tilde{x}+1)V_1(\tilde{x}+1),$$
$$2\tilde{y}V_2(\tilde{y})<(\tilde{y}-1)V_2(\tilde{y}-1)+(\tilde{y}+1)V_2(\tilde{y}+1),$$

and these are satisfied by the convexity of $zV_i(z)$. This means that $(\tilde{x}, \tilde{y})$ is the socially optimal solution.□

The proposition above says that each individual's choice of path is 'compatible' with social optimality for socially beneficial congestion cost functions. That is, using real-time information is beneficial to each individual and to the society as a whole when (V_1, V_2) is socially beneficial. We now extend our definition for an individual's value of real-time information to the case of a society. As in Lemma 3, denote $x^*(m)$ as the optimal flow for path 1 given that the total number of cars trying to use either path 1 or 2 is m. In reality, m is not deterministic and thus let M be the random number of cars trying to use one of paths 1 and 2. Now define the value of real-time information for the society as follows:

$$V=E_M E_{X|M}[XV_1(\mathrm{X})+(\mathrm{M}-\mathrm{X})\mathrm{V}_2(\mathrm{M}-\mathrm{X})]$$
$$-\mathrm{E_M}[\mathrm{x}^*(\mathrm{M})\mathrm{V}_1(\mathrm{x}^*(\mathrm{M}))+(\mathrm{M}-\mathrm{x}^*(\mathrm{M}))\mathrm{V}_2(\mathrm{M}-\mathrm{x}^*(\mathrm{M}))].$$

We can easily see that the value of information defined above is positive as follows: for socially beneficial(V_1, V_2) and under the convexity of $zV_i(z)$, given m, real-time information induces socially optimal cost of $x^*(m)V_1(x^*(m)) +(m-x^*(m))V_2(m-x^*(m))$ from individuals' path choice. We thus have

$$xV_1(x)+(m-x)V_2(m-x)>x^*(m)V_1(x^*(m))+(m-x^*(m))V_2(\mathrm{m}-\mathrm{x}^*(\mathrm{m})),\ \text{for } x\neq x^*(m).$$

$$\sum_{x=0}^{m}[xV_1(x)+(m-x)V_2(m-x)]P(X=x\,|\,M=m)>$$
$$\sum_{x=0}^{m}[x^*(m)V_1(x^*(m))+(m-x^*(m))V_2(m-x^*(m))]P(X=x\,|\,M=m).$$

$$\sum_{m=0}^{\infty}\sum_{x=0}^{m}[xV_1(x)+(m-x)V_2(m-x)]P(X=x\,|\,M=m)P(M=m)>$$
$$\sum_{m=0}^{\infty}\sum_{x=0}^{m}[x^*(m)V_1(x^*(m))+(m-x^*(m))V_2(m-x^*(m))]P(X=x\,|\,M=m)P(M=m).$$

But we cannot expect each individual's choice for his own benefit to be compatible with the social optimality in general(especially for congestion cost functions which are not socially beneficial). In this case a government agency might intervene to induce the socially optimal solution. In the general case, we may levy 'tolls' for using a certain path and thus adjust the cost function perceived by an individual. With the adjusted cost function with the right amount of toll, each individual makes a choice on the path and constructs the socially optimal allocation of cars to each road. In reality, levying 'toll' can be effected as follows. One possibility is to manipulate traffic lights through the roads. By doing this we can change the time and the cost for using that particular road. Another possibility is to actually levy tolls for using tunnels or some segments of a road and induce the customer flow to the best of social utility.

3. Case Where Departure Time Is a Decision Variable

Now we deal with the case where we have a predetermined time at which we should deliver products to a customer. You can think of a parts delivery in JIT manufacturing system. In this setting we can choose our departure time for delivery and we have two kinds of cost: delay cost and early arrival cost. We suppose that cost of c_d incurs per unit time of delayed arrival with respect to the schedule. And c_e is the cost incurring for a unit time of earlier arrival than scheduled. When a truck company is required to deliver parts for JIT of a factory, not only c_d but also c_e plays a role in optimization. We define the critical fractile $\eta = c_d/(c_d+c_e)$.

Suppose that we depart w(decision variable) hours before scheduled time point t and let us denote ξ as a random variable representing the time spent to arrive at the destination.

Then for a given w, the cost incurred is $c_e(w-\xi)^+ + c_d(\xi-w)^+$. We now are trying to choose w^* which minimizes the expected cost of $c_e(w-\xi)^+ + c_d(\xi-w)^+$. Let F, G, and H be distribution functions for X, Y, and $Z=X \wedge Y$ respectively and these random numbers have the range of $[0, T]$. Then we can derive that $H(z)=F(z)+G(z)-F(z)G(z)$.

Denote the expected cost along path 1 when we depart w hours before schedule as $C^X(w)$ and

$$C^X(w)=c_e\int_0^w(w-\xi)f(\xi)d\xi+c_d\int_w^\infty(\xi-w)f(\xi)d\xi.$$

This takes the similar cost function in newsvendor problem in inventory control. The optimal departure time for a company using path 1, W_1, is from $F(W_1)=\eta$. When we use real-time information and choose a path accordingly, the time for arrival is Z. And let W_0 denote the optimal departure time for a company using real-time information on traffic conditions. Then as before, W_0 satisfies $H(W_0)=\eta$. Since $H(z)\geq F(z)$ for all z, we know that $W_1\geq W_0$. We now compare the optimal cost when we lack real-time information with that from using real-time information.

Proposition 2 $C^X(W_1)>C^Z(W_0)$ *holds if and only if*

$$\eta/(1-\eta)>\kappa/\phi \;(**)$$

where

$$\phi=\int_{W_1}^T uf(u)du-\int_{W_0}^T uh(u)du,$$

$$\kappa=\int_0^\eta F^{-1}(u)du-\int_0^\eta H^{-1}(u)du=\int_0^{W_1}uf(u)du-\int_0^{W_0}uh(u)du.$$

Proof:

$$\begin{aligned}C^X(W_1)-C^Z(W_0)&=c_e[\int_0^{W_1}F(\xi)d\xi-\int_0^{W_0}H(\xi)d\xi]+c_d[\int_{W_0}^T H(\xi)d\xi-\int_{W_1}^T F(\xi)d\xi]+c_d(W_0-W_1)\\&=c_e[\mu z-\mu x+\int_{W_0}^T H(\xi)d\xi-\int_{W_1}^T F(\xi)d\xi]+c_d[\int_{W_0}^T H(\xi)d\xi-\int_{W_1}^T F(\xi)d\xi]+c_d(W_0-W_1)\\&=c_e(\mu z-\mu x)+c_d(W_0-W_1)+(c_d+c_e)(\tau+\phi)\\&=c_e(\mu z-\mu x)+(c_d+c_e)\phi=c_d\phi-c_e\kappa\end{aligned}$$

where $\tau=(W_1-W_0)\eta$. Note that $\phi+\kappa=\mu x-\mu z$. Thus the condition $C^X(W_1)>C^Z(W_0)$ is the same as $c_d\phi>c_e\kappa$ or equivalently $\eta/(1-\eta)>\kappa/\phi$.□

The condition (**) is not satisfied in general. This result can be explained as follows. Even though we can shorten our delivery time using real-time information on traffic conditions, the optimal cost by choosing optimal departure time is not necessarily smaller than that of without the information case. Even without real-time information we can adjust our departure time and thus can possibly have less expected cost. But with real-time information, we have surplus time of W_1-W_0 which can be used profitably. Thus when we incorporate the value of extra time(W_1-W_0) due to real-time information, it is likely that we are better off with real-time information even when (**) is not satisfied. Both an example where (**) is satisfied and a case not satisfying (**) are given.

Example 3: $X \sim Exp(\lambda)$ and $Y \sim Exp(\mu)$. Then $Z \sim Exp(\lambda+\mu)$. In this case, denoting $\bar{\eta}=1-\eta$, we get $W_1=-\ln\bar{\eta}/\lambda$ and $W_0=-\ln\bar{\eta}/(\lambda+\mu)$. And $\phi=[\mu(\bar{\eta}-\bar{\eta}\ln\bar{\eta}]/[\lambda(\lambda+\mu)]$ and $\kappa=[\mu(\eta+\bar{\eta}\ln\bar{\eta})]/[\lambda(\lambda+\mu)]$. The condition (**) now becomes

$$\frac{\kappa}{\phi}=\frac{\eta+\bar{\eta}\ln\bar{\eta}}{\bar{\eta}-\bar{\eta}\ln\bar{\eta}}=\frac{\eta+\bar{\eta}\ln\bar{\eta}}{1-\eta-\bar{\eta}\ln\bar{\eta}}<\frac{c_d}{c_e}=\frac{\eta}{\bar{\eta}}.$$

This condition is satisfied since $(c_d+c_e)\,\bar{\eta}\ln\bar{\eta}<0$.

Example 4: Let $X \sim U[0, 1]$ and $Y \sim U[0, 1]$. Then $\phi=-1/2+\eta-\eta^2/2+2(1-\eta^2)/3$ and $\kappa=2/3-\eta+\eta^2/2-2(1-\eta)^2/3$. The condition (**) now becomes $\eta(\eta-1)^2(9\eta-8)<0$. Therefore, for $\eta\in(8/9, 1)$, the condition (**) is not satisfied.

4. Practical Implications

Throughout this paper we studied the congestion cost functions from two perspectives: an individual and the society as a whole. For an individual's perspective, we were interested in the following: how much benefit will the driving customer get if he gets the real-time information on the traffic conditions on the roads which lead him to the destination? How much cost will a trucking company save by using the real-time information on the traffic conditions on the roads they are using? But the social perspective is not less important than the private perspective. This is especially true when the government considers the investment on the information system which will give consumers the real-time information on the traffc conditions. The reason is that the government should now consider the social welfare improvement by introducing the information system, not just a person's cost reduction. In analyzing the investment on the information system, the government or other appropriate agency should consider the cost and the benefit it will induce when implemented. The result of Proposition 1 gives us the benefit from the information system for the society as a whole and thus helps the government set up an investment policy. As mentioned above we deal with the implications for both private and social sectors.

4.1 Private Sector

By using the previous analysis, we can calculate the mean saving in cost which incurs from using the real-time information on the roads. If a system which will give us the real-time information requires less cost(including both the investment cost and usage cost) than the cost saved from the system, we had better go for the system. Thus our analysis will help us decide the price for those information systems using satellites giving the real-time information on the roads. GPS(Global Positioning System) would be a good example for this application.

And the author observed a private company which gives transportation service to the customers from/to airports using TRS(Trunked Radio System). Drivers in each limo communicate interactively with the central office. By using the communication equipment, they give to the central office the real-time traffic condition on the road which they are currently using. This information on the roads is dispatched through the communication channel to those drivers who might use the roads and thus help them choose the less congested path to the destination. And the driver who gave the information on the traffic conditions also gets the real-time information on the next roads from the central office. And this information is received from the other driver who observed the situation previously. In this system, the central office works as a server which updates the information on the traffic conditions on roads and gives the appropriate information to the driver needing it. The cost saving and customer satisfaction increment from arriving at the destination faster will be measured by the analysis studied in this paper.

4.2 Public Sector

To understand the social perspective, let us consider the following situation. There are two paths(path 1 and path 2) for a customer between which he can choose for driving to the destination. By getting the real-time information on the traffic conditions on paths 1 and 2, he will choose the road which will lead him to the destination faster. This is the decision made by a person from his own perspective. He does not take into account whether his decision will improve the social benefit as a whole. That is, his decision is based solely on his cost reduction, not on its impact on social cost due to traffic congestion. But his choosing the faster path may alleviate the social cost. This is because the cost in the traffic system as a whole might have become even larger if he had chosen the slower path due to lack of information and aggravated the already congested path.

When a government agency considers an information center which gives real time information on traffic conditions, it should take into account the society's cost reduction which the information center will induce. Even though an individual decides a path for his own utility, the decision affects the traffic system as a whole and these impacts should be the criterion for the government agency. Though it is possible for private companies or individuals to invest on the information system and get the benefit of using real time information, there should be cases where it is better from cost / benefit perspective to invest the system for the society as a whole.

The information systems, which are widely used for giving traffic information, are radio broadcasting channels focused on traffic information. They give information on traffic and other accidents, and traffic conditions of several major roads in a city. By tuning on the traffic braodcasting channel, the drivers get the information on the roads and can respond by taking less congested routes to the destination. The major problem in this system is that the information flow is not based on individual demand and it is almost one way. A driver can not get the information when needed on the traffic condition of the road which he considers to enter. The order of the roads covered in the radio is set up and can not be changed for an individual's utility.

Thus we have to consider the information systems which are somewhat interactive and selective in the sense that individuals can get the information on the roads in which they are interested in. This information is truly real time since the customers can get it when needed. We can think of an information center which updates the information on the traffic conditions of roads in a city by the real time feedback from member of drivers or closed circuit TVs. They set up codes for each segment of roads in the city and respond to the requests from the customers by giving the current information on the traffic condition of the road segment for their benefits by ARS

(Automated Response System).

By giving the information on the traffic conditions on the roads in the city, the information center can indirectly guide the individuals to less congested roads and thus improve their benefits. Also as studied previously, the information, in many cases, will give us resource pooling effect in the sense that all the roads are somewhat equally used and thus total social benefit increases.

5. Concluding Remarks

In this paper, we showed that there would be improvement in benefit by using real time information on traffic conditions. By using real time information on traffic conditions of each route, an individual could choose the less congested path and thus get to the destination faster than otherwise. With real time information, we could locate and choose less congested path to the destination, which does not necessarily incur less time in the mean. Real time information makes it possible to utilize the paths for a customer's benefit even though they are inferior in the mean time. This kind of benefit improvement was from individual point of view. Real time information could improve not only an individual's benefit but also the society's benefit as a whole. An individual's choosing less congested path due to real time information could alleviate the society's congestion cost since it prevents the already congested road from being exacerbated by incoming customers.

Although we analyzed a traffic system which has multiple routes to the destination, the methodology can be applied to a general queueing system. Consider the queueing system where there are two servers in the system. Without any information on the queue state, we basically have two independent queues. If we have the real-time information on the queue states and can route arriving customers to less congested queue, we can utilize the service capacity more efficiently and thus can save the mean throughput time substantially(Nam[1993]). Thus our method can be corresponded to the dynamic scheduling(routing) problem in queueing system with multiple queues and servers.

A real traffic system can be represented by a network, which is a more general form than that covered in this paper. A general queueing newtork is well-known for its analytical complexity(Harrison[1988]). As asserted previously, one of our focuses is on an individual(clearly having one departure node and one destination node) whose time is saved through real-time information. This means that an individual can choose for its benefit one of the multiple paths from a departure node to the destination by using real-time information. Our model can also be generalized to the case where an individual has the option of not entering the traffic system as well.

Throughout this paper, we implicitly assumed that once we knew the traffic conditions in each route these wouldn't change until an individual is served(i.e. arrive at the destination). In the future research, we should try to generalize this assumption. And we should also incorporate adequate cost function for introducing real time information system as well. In the future, we should empirically study the waiting cost function of each individual and derive the value of real-time information.

Appendix

A.1 Socially beneficial Congestion Costs-Continuous Case

We give corresponding lemmas for Lemma 2 and Lemma 3 in the case of $V_i : R^+ \rightarrow R^+$. We assume that $zV_i(z)$ is convex as before and V_i are twice continuously differentible for $i=1, 2$.

Lemma 4 *The necessary and sufficient conditions for* (V_1, V_2) *to be socially beneficial are: for any* (x, y) *such that* $V_1(x)=V_2(y)$, *we have*

$$xV_1'(x)=yV_2'(y).$$

For this lemma, define $K(x, y)=xV_1(x)-yV_2(y)$ and use $k(x, y)=\frac{d}{dx}(xV_1(x)) - \frac{d}{dy}(yV_2(y))$, which is the directional derivative of K along $[1, 1]'$.

Lemma 5 *When there are* m *customers in the society, the condition for* x^* *to be a socially optimal flow amount for path 1 is:*

$$V_1(x^*)+x^*V_1'(x^*)=V_2(m-x^*)+(m-x^*)V_2'(m-x^*).$$

A.2 Examples of Socially Beneficial Congestion Costs

We now give five examples of socially beneficial congestion cost functions. The last example is the case of non-socially beneficial cost functions.

Example 1 : $V_1(x)=ax^k$ and $V_2(y)=by^k$ where $a, b > 1$.

Example 2 : $V_1(x)=c^x$ and $V_2(y)=d^y$ where $c, d > 0$.

Example 3 : Any symmetric congestion cost functions, that is, $V_1(x)=V_2(x)$ for all x.

Example 4 : $V_1(x)=2\sqrt{x}$ and $V_2(y)=\sqrt{y}$. In this example, the reader should note that V_i are not convex functions but they are socially beneficial.

Example 5(Queueing) : We take mean waiting time for a queueing system as a congestion cost function. That is,

$$V_i(\lambda_i)=W_i(\lambda_i)=\frac{1}{\mu_i-\lambda_i}$$

for $i=1, 2$.

Example 6 : Let $V_1(x)=x$ and $V_2(y)=y^2$. At (10, 3), we have $V_1(10)>V_2(3)$. But $(11)V_1(11)+3V_2(3)=148<10V_1(10)+4V_2(4)=164$.

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