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Risk Sharing and Interbank Market Fragilities*

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ABSTRACT

Risk sharing among banks helps them diversify idiosyncratic risks, but their interbank borrowing costs can become more volatile and bring financial fragility. Banks facing liquidity shortages need to pay an extra cost of credit when their lenders have bargaining powers, which depends on the likelihood of fire-sale and the fire-sale price discount. Risk sharing can decrease likelihood of liquidity shortage and lower the borrowing cost. However, the fire-sale discount increases, since joint distress arises and more assets are liquidated simultaneously. Though the interbank borrowing cost decreases with risk sharing, it may become more sensitive to changes in aggregate uncertainty.

Keywords: risk sharing, interbank market, network, financial crisis

1. INTRODUCTION

Financial innovations provide banks various tools to transfer their idiosyncratic risks for diversification. Such risk sharing enables banks to reduce tail risks, which would make the financial system

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more robust (Allen and Gale 2000; Zawadowski 2013). However, there also exists a downside when the network becomes more intertwined; while the likelihood of individual failure becomes smaller, when a failure happens, it tends to be systemic, as many networks are interconnected (Freixas, Parigi, and Rochet 2000; Leitner 2005; Brusco and Castiglionesi 2007; Castiglionesi, Feriozzi and Lorenzoni 2010; Wagner 2009; Ibragimov, Jaffee, and Walden 2011). Hence, the system can become more robust ex ante through the diversifications, but once a tail event occurs, it becomes systematic. Therefore, risk sharing is beneficial *ex ante*, but can be detrimental *ex post*.

This argument implies that systemic distress, while more critical, becomes a rare event as more risks are shared and better diversified. Thus, though damage from a potential crisis could become severer, it may be of secondary importance ex ante, since it is literally a rare event.[1] On the contrary, this paper suggests that risk sharing may increase financial fragility even *ex ante* without the actual realization of the tail event.

I focus on an economy with two specific frictions: (1) interbank lending markets that are not perfectly competitive (thus, lenders have bargaining power), and (2) illiquid secondary asset markets (possibility of fire-sale discount). In the model, there are two types of banks that trade in the interbank market, borrowers and lenders. Borrowers face interim idiosyncratic liquidity risks with random size, and need to inject a certain amount of liquidity if hit by the shock. Since cash hoarding is costly, the borrower tries to borrow liquidity from its lender. My model focuses on this interbank borrowing cost that a liquidity-lacking bank needs to pay to secure a liquidity provision from its lender, and analyzes how risk sharing affects volatilities of this cost.

The first friction comes from relationship lending in the interbank market.[2] This implies that a borrower cannot easily replace its current lender with others; thus, the lender has bargaining power

1) In October 2008, Alan Greenspan testified at Congress that “We have to recognize that this is almost surely a once-in-a-century phenomenon, and in that regard, to realize the types of regulation that would prevent this from happening in the future are so onerous as to basically suppress the growth rate in the economy and … the standards of living of the American people.”

2) See Bech and Atalay 2010; Cocco, Gomes, and Martins 2009 on the evidences of a lending relationship in the interbank market.

over the borrower (see Bech and Klee 2011, for the empirical evidence). The lender is aware of this hold-up cost, that is, the borrower will face a difficulty if it refuses to lend. This becomes the source of the lender's rent. For simplicity, I assume that a borrower can only borrow from a certain lender. The size of this rent then depends on the scale of the second friction, liquidity of the secondary market. The borrower has to liquidate its long-term assets for a discounted price if it fails to borrow from the lender, and anticipating this, the lender can charge a higher cost for its interbank lending to extract the rent.

If the borrower tries to borrow in the spot market after the liquidity shock is realized (ex post borrowing), the hold-up problem becomes more severe, because the lender tries to extract all the economic profit regardless of the size of the shocks. The borrower thus makes a credit line contract ex ante (before the shock arises) to avoid the lender's ex post opportunism and reduce the rent. Note that the lender's rent is still non-zero and, it now comes from the *expected* fire-sale loss that the borrower would suffer if the lender refuses to provide a loan. The expected loss is the product of the following two factors: (1) the likelihood of the potential fire-sale event, and (2) the amount of fire-sale discount, which increases in the amount of liquidated assets. In sum, the interbank borrowing cost that reflects the size of the lender's rent depends on

likelihood of liquidity distress × *fire-sale discount in that distress event*

when the lender refuses to provide liquidity to the borrowers.

In this case, the borrowers can lower the borrowing cost through risk sharing among themselves, because they can diversify idiosyncratic liquidity risks and compress the size of the first factor—the distress event becomes less likely with diversification. The tradeoff here, as argued in the literature, is that once the distress event arises, it becomes systemic, since many entities are interconnected and jointly affected. In our setup, this aspect is captured by the larger second factor—the fire-sale asset price becomes very low when the episodic distress occurs, because many entities would be jointly distressed, and more assets would be liquidated simultaneously. Facing this tradeoff, the borrowing banks choose to share risks and become interconnected if the decrease in the first factor is large enough to offset the increase in the second factor. Risk sharing here lowers the borrowing cost by suppressing the speculative lender's rent.

Our model also suggests that this risk sharing arrangement can bring a novel source of financial vulnerability—the interbank market can become more fragile when aggregate uncertainty increases, e.g., higher likelihood of a tail event, when banks are more interconnected. Suppose that the right tail of the liquidity shock distribution becomes longer as the aggregate uncertainty increases. This implies that the first factor became larger. In this case, the *sensitivity* of the lender's rent can become very different with and without risk sharing, because of the difference in the second factor, the fire-sale discount. If risks are not shared, the effect of this change on the lender's rent is relatively small—while the future fire-sale event becomes more likely, not many assets should be liquidated at the same time implying a low fire-sale discount, which results in a small change in the lender's rent. If risks are shared, the lender's rent increases rapidly; when the tail event becomes more likely, the lender's rent rises faster because of the larger price discount from the joint distress and more fire-sales. As the sensitivity of this rent is equivalent to the sensitivity of the interbank borrowing cost, risk sharing makes the interbank market more vulnerable when aggregate uncertainty fluctuates.

We may then consider a case in which banks instantly implement risk sharing to lower their borrowing cost, but the cost becomes more sensitive to the aggregate uncertainty fluctuations, and gets even higher than that without risk sharing once the aggregate uncertainty increases. When there is an upper bound in this cost in the manner of Stiglitz and Weiss (1981) with corresponding credit rationing, there arises a real effect from this fragility as aggregate uncertainty increases; aggregate output drops with credit rationing since the borrower cuts down investment and hoards liquidity. This causes a welfare loss to the aggregate economy, which may have been avoided if there were no (or less) risk sharing.

In terms of welfare, the banks may share risks "excessively" in pursuit of low borrowing cost, which eventually brings fragility. From the social planner's perspective, aggregate welfare is maximized when there is no potential credit rationing. Note that the cost of interbank borrowing itself is a mere transfer, and not of concern for the social planner unless it distorts resource allocation. Banks, on the other hand, do care about their borrowing cost. If the instant benefit of cheaper borrowing cost achieved by risk sharing is greater than the cost of potential credit rationing, they choose to share risks and introduce fragilities, although this is not desirable from the social welfare perspective.

This paper is mainly related to two strands of literature. The first is based on the bank liquidity portfolio problem and interbank market stress. The liquidity provision problem of banks is studied in Diamond and Dybvig (1983), Allen and Gale (1994, 1998), and Diamond and Rajan (2005). Battacharya and Gale (1986) study liquidity provisions in the interbank market with private information, and Heider, Hoerova, and Holthausen (2009) provide a model of interbank market breakdown with severe asymmetric information problems. Holmström and Tirole (1998, 2008), and Bolton, Santos and Scheinkman (2011) study an optimal mix of different sources of liquidity. In Holmström and Tirole, cash hoarding and a line of credit are equivalent as a liquidity buffer, since the lender does not speculate and always breaks even (supplied by the government). In Bolton et al., banks may meet their liquidity demand with either cash hoarding or asset sales. Their focus is on the effect of asymmetric information on the potential buyer's liquidity provision, while our focus is on the potential lender's strategic behavior (as in Acharya, Gromb, and Yorulmazer (2012), and Diamond and Rajan (2011)), and how risk sharing can affect these speculative motivations. Afonso, Kovner, and Schoar (2011) study the federal funds market during the 2008 crisis, and find that the interbanking market did not completely freeze up, but the lending rates did increase, and the increments varied across banks depending on a borrowing bank's characteristics, which my model also predicts.

The second related strand of literature is based on the aggregate effects of financial networks, homogenization, or risk sharing. Systemic robustness increases with risk sharing (Allen and Gale (2000), Zawadowski (2013)), but the distress becomes systemic once it arises (Freixas, Parigi, and Rochet 2000; Leitner 2005; Brusco and Castiglionesi 2007; Castiglionesi, Feriozzi and Lorenzoni 2010; Wagner 2009; Ibragimov, Jaffee, and Walden 2011). I argue that even *ex ante*, risk sharing can generate a certain type of fragility.

The motivations of risk sharing (or homogenization) are different across papers. Acharya and Yorulmazer (2007, 2008b) suggest that the "too many to fail" argument leads investors to become homogenized, so that they get bailed out when in trouble. In Acharya and Yorulmazer (2008a), a possible information contagion induces the banks to herd. In the literature, the sources of negative externalities of risk sharing are often exogenous. Acharya (2001), Wagner (2009), and Ibragimov et al. (2011) assume some social costs of joint failure. In Castiglionesi, Feriozzi and Lorenzoni (2009), excessive risk sharing comes from pecuniary externalities. In my model, banks share their risks, not only to diversify their interim shocks, but also in seeking cheap credit, and this can impose a negative externality by causing fragility and inefficient resource allocation.

2. THE MODEL SETUP

Consider a three period (t = 0,1,2) economy with a single consumption good. There are two types of agents (banks) in this economy, borrowers and lenders. For simplicity, we assume that there are two borrowers (A and B) and one lender. All of them are risk neutral, and only consume at t = 2.

2.1 Borrowers

The borrowers are liquidity-lacking banks facing interim liquidity risks in the interbank market.[3] They are endowed with 1 unit of initial goods. At t = 0, they can invest in either of two assets (projects), a short-term asset or a long-term asset. The short-term asset (liquid asset) is a storage technology, and one unit of the goods stored produces one unit in the next period, which can act as a liquidity cushion for potential interim liquidity requirements. The long-term asset takes two periods to mature. One unit of the goods invested in the long-term asset at t = 0 produces $R > 1$ at t = 2. It produces nothing at t = 1, but can be sold in the secondary market at an endogenous liquidation price of P per unit. The borrower makes an optimal portfolio decision at t = 0, investing y in the short-term asset and $1 - y$ in the long-term asset, given the interim liquidity risk described below.

3) While we focus on the interbank market, our setup can also apply to non-bank borrowers.

At $t = 1$, borrower i (= A, B) experiences an idiosyncratic interim shock $\tilde{\rho}_i$, which stands for an amount of required liquidity injection.[4)] For simplicity, I assume that the shocks are exclusive, such that only one of the two borrowers can receive a positive shock at a time.[5)] To be specific, either A or B receive a liquidity shock with a probability of p (< 1/2) each, and neither entity experiences any shock with probability $1 - 2p$. When hit by a shock, the size of the potential shock follows $\tilde{\rho} \sim U[0, \bar{\rho}]$. Thus, the ex ante distribution of an individual liquidity shock can be represented as

$$\tilde{\rho}_i = \begin{cases} 0 \text{ with prob } 1 - p \\ \tilde{\rho} \text{ with prob } p, \text{ where } \tilde{\rho} \sim U[0, \bar{\rho}] \end{cases} \tag{1}$$

I interpret $\bar{\rho}$ as a measure of aggregate uncertainty, where higher $\bar{\rho}$ implies higher aggregate uncertainty in this economy.[6)] I assume $\bar{\rho}$ < 1, such that the size of the shock is less than the insider's initial endowment.

Let ρ be a realization of the liquidity shock $\tilde{\rho}$. The borrower can meet this liquidity demand using two sources of liquidity: its own short-term asset hoarding, and a loan from the lender (interbank lending). If the borrower cannot secure enough liquidity to pay ρ, it is in financial distress, and is forced to liquidate (fire-sale) its long term asset $1 - y$ in the secondary market at a price of P per unit. The early liquidation price P decreases in the amount of total assets sold in the secondary market.

2.2 Interbank lending from the lender

Before the portfolio decision at $t = 0$, the borrowers can approach the lender for a loan commitment contract to secure a credit line. A credit line contract is characterized by a loan limit amount and an (endogenous) interest rate r for the actual loan used. If the lender accepts, the borrower can borrow up to the specified limit at $t = 1$ after the shock realizes, and pays $r \cdot \rho$ at $t = 2$ upon borrowing ρ at

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4) This can come from unused loan commitment, liquidity, and credit enhancement.

5) We can assume i.i.d. shocks instead of exclusive shocks, but the main implications of the model remain the same.

6) This is for tractability. More intuitive measures are FOSD or mean-preserving spread of $\tilde{\rho}$.

t = 1.[7] I consider r as a measure of borrowing cost. Alternatively, the borrowers can borrow from the lender in the spot market at t = 1 after the shock realizes, instead of contracting an ex ante credit line.[8]

2.3 Lender's speculation and rent

I assume that relationship lending is critical in the interbank market, and the borrowers can only borrow from the lender in the interbank market.[9] Thus, if the lender refuses to lend, the borrower, who cannot meet the interim liquidity demand, has to liquidate its assets early, at a loss. This gives the lender some bargaining power over the borrowers, from the hold-up problem (Rajan 1992).[10] For simplicity, I assume that the lender can also purchase the borrower's asset in the secondary market, given an exogenous downward sloping demand curve. This is equivalent to allocating 100% bargaining power to the lender in the Nash bargaining setup.

The lender is endowed with M (> $\bar{\rho}$) units of the good, and has 3 investment alternatives: (i) lending to the borrowers in the interbank market; (ii) buying fire-sale assets in the secondary market; and (iii) storage. The lender cannot initiate the long-term project by himself without the proper origination technology. However, he can run the long-term project upon an acquisition in the secondary market at t = 1, and still acquire R per unit at t = 2.

Without the second option, the lender may simply try to break even when lending in the interbank market. But the lender can rather choose to become a speculative buyer, if this option is more profitable (collecting fire-sale assets at a discounted price). In the t = 1 spot market, it will refuse to lend to the borrowers if it can collect higher profits by letting the borrowers fall into distress. The borrower thus has to pay extra (cost of credit) to induce the lender to

7) I assume that this credit line contract is ex-post enforceable. See Boot, Greenbaum, and Thakor (1993) for the theoretical model based on reputation concerns.

8) I will later show that spot borrowing is dominated by the credit line contract; thus they don't arise in equilibrium.

9) This can be due to the high switching cost from asymmetric information and asset specificity, for instance.

10) While the lenders in our model are opportunistic, they would behave more cooperatively if, for instance, reputation or long-term relationship matters.

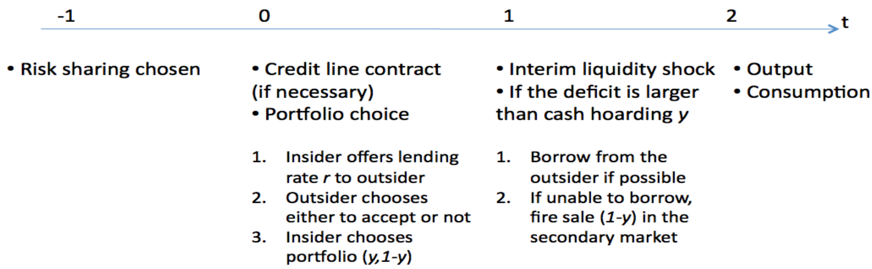

Figure 1. Timeline

provide liquidity in this case. However, the borrower can avoid this ex-post opportunism and secure a loan provision by contracting a line of credit ex ante at $t = 0$. Again, the lender may refuse to accept this credit line offer if it has a better outside option: waiting for the borrower's distress and collecting the discounted fire-sale assets in the secondary market. Thus, the extra cost of credit still needs to be paid ex ante at $t = 0$ if the lender's expected profit (rent) is nonzero, even for the credit line contract. Note that we get the same result in the Nash bargaining setup, even without the lender's purchasing option, since the borrower is worse off if the lender refuses to lend, which is the source of the lender's bargaining power, as in our speculative purchasing setup.

The timeline is summarized as follows (Figure 1). At $t = 0$, the borrowers first approach the lender for a credit line contract if necessary. The lender then chooses either to accept the offer or not. Next, the borrowers choose their investment portfolios $(y, 1 - y)$, which are not contractible. At $t = 1$, a liquidity shock realizes, and the borrowers experiencing the liquidity shock try to borrow from the lender if necessary. Distressed borrowers long-term assets are liquidated in the secondary market if they fail to secure enough liquidity. At $t = 2$, output is produced, and agents consume.

3. LIQUIDITY PROVISION AND COST OF CREDIT

I now solve for the cost of credit in the interbank market and ex ante liquidity provision of the borrower. For simplicity, I assume that the lender is the only potential buyer in the secondary market, due to the specificity and complexity of the underlying long-term asset.[11] I further assume that all agents are price takers in the secondary market.

The asset price in the secondary market can deviate from the fundamental value with limited market liquidity, and this wedge becomes larger as more assets are sold. In this case, the lender may be able to collect nonzero profit in the secondary market when there is a fire-sale, which is increasing in the amount of asset liquidation. This gives it an incentive to become a “vulture” buyer instead of being a “friendly” relationship lender.

For tractability, I assume that the asset price is given by cash-in-the-market pricing, as in Allen and Gale (1994, 1998), when the size of asset liquidation is large. Given the lender’s endowment M, the asset price P deviates from its fundamental value R when the total amount of long-term assets liquidated in the secondary market, denoted by Q, is large, such that

$$P = \min\left\{R, \frac{M}{Q}\right\}, \tag{2}$$

where $Q = (1 - y)$ in our case, since only one of the two borrowers will be distressed at a time. If $P < R$, the lender gets a positive profit $(R - P)$ per unit as a buyer. Thus, the borrowers have to compensate the lender for this “rent” if they wish to borrow from the lender, and this rent affects the credit cost in the interbank market.

Recall that a line of credit contract is characterized by a credit limit and an accompanying interest rate r. In the appendix, I show that the borrower prefers having a credit line contract to fully protect itself from potential early liquidation, to borrowing in the spot market at $t = 1$ (or not borrowing at all). Intuitively, this is because the hold-up cost becomes larger at $t = 1$ once the shock realizes; the lender will always refuse to lend unless he can extract the maximum amount of rent, regardless of the size of shocks. The borrower can reduce this cost if it succeeds in securing a loan commitment at $t = 0$. If the lender refuses to provide a loan commitment, it can hoard some short-term assets by itself, which suppresses the lender’s speculative motive. The borrower wishes to set r high enough to have the lender agree to lend, but as low as possible to minimize the

11) This is for simplicity. All we need is a downward sloping asset demand curve in the secondary market.

borrowing cost.

Hence, I focus on deriving an equilibrium interest rate (cost of credit) r^* of a credit line contract which the borrower offers to the lender at $t = 0$, and the borrower's portfolio decision y^*. In equilibrium, (y^*, r^*) maximizes the borrowers expected payoff, while at the same time inducing the lender to supply a line of credit in the interbank market.

I solve the borrower's optimal problem (y^*, r^*) taking the following steps. First, I calculate the lender's rent (off-the-equilibrium payoff) when he refuses to be a lender and chooses to collect fire sale assets (if any). Notice that the borrower has to pay this rent to the lender as an additional cost in order to induce him to lend. Given the size of this rent, I next solve for the optimal loan contract (r^*) and the optimal portfolio decision (y^*) for the borrower. I focus on the symmetric equilibrium and treat the borrowers A and B in an equal way.

3.1 Lender's expected profit (rent) when refusing to lend

We first derive the borrower's own $t = 0$ liquidity hoarding, denoted by y^V, when the lender refuses to lend. We then derive the lender's expected profit when refusing to lend, given this borrower's optimal response. Given the size of the lender's rent, we then solve for the borrower's optimal portfolio problem as well as the cost of borrowing to induce the lender's agreement.

When the lender refuses to provide liquidity, the borrower i's payoff with short-term asset holding y, denoted as $U_i^V(y)$, is equal to

$$U_i^V(y) = [(1 - y)R + y] \times \Pr(\tilde{\rho}_i \leq y) + [(1 - y)P + y] \times \Pr(\tilde{\rho}_i > y) - E(\tilde{\rho}_i) \quad (3)$$

where $\tilde{\rho}_i$ follows (1), and $P = \min\{R, \mathrm{M}/(1 - y)\}$ taken as given. The superscript V stands for the lender as a "vulture buyer," and the borrower has to sell if the shock $\tilde{\rho}$ is larger than his liquidity cushion y without any loan provision. Since this is strictly concave in y, we can pin down unique $y^V \in [0, 1]$ such that

$$y^V = \mathrm{argmax}_y U_i^V(y) \quad (4)$$

where y^V is the optimal liquidity hoarding when there's no outside liquidity available.

Now, let $\Pi^V \equiv \Pi^V(y^V)$ be the lender's expected profit (rent) in this case, given that the borrower hoards y^V as his own liquidity cushion (optimal response). We get

$$\Pi^V(y^V) = (1 - y^V) \times (R - P) \times Pr\,(\tilde{\rho} > y^V) \times 2p$$

$$= (1 - y^V) \times (R - \min\{R - \frac{M}{1 - y^V}\}) \times (1 - \frac{y^V}{\bar{\rho}}) \times 2p \qquad (5)$$

where the first term is the amount of asset sold, the second term is profit margin per unit of asset, and the rest is the likelihood of buying opportunity.

3.2 Optimal Loan Contract and Portfolio Decision

Given the lender's rent Π^V given by (5), we now solve for the borrower's optimal line of credit contract r^* and optimal investment portfolio y^*. Since the lender expects to get positive profit Π^V by refusing to lend, the borrower has to compensate at least Π^V in expectation as an extra cost of credit, in order to borrow. Thus, optimal investment y^* and interest rate r^* are such that (i) the lender expects to receive no less than Π^V as an interest payment, and (ii) the borrower maximizes his expected payoff.

Given r and y, the borrower i's expected payoff is characterized as

$$U_i^L(y, r) = (1 - y)R + y - rE[\max(0, \tilde{\rho}_i - y)] - E(\tilde{\rho}_i) \qquad (6)$$

the superscript L stands for the loan providing lender. As a first step, we can easily derive the optimal y^* as a function of r ($y^* = y^*(r)$).

Lemma 1. (Optimal liquidity hoarding given r)

For a given interest rate r, with $\bar{r} = \dfrac{R - 1}{p}$

- *if* $r \leq \bar{r}$, *then* $y^*(r) = 0$. $U_i^L(y(r);\, r)$ is independent of $y(r)$, strictly decreasing in r.
- *if* $r > \bar{r}$, *then* $y^*(r) = \bar{\rho}\left[1 - \dfrac{(R-1)}{rp}\right]$.

This implies that when interbank borrowering is not very costly, the borrower will not rely on costly liquidity hoarding so as to invest more in the long term asset. Liquidity hoarding increases as the cost of interbank borrowing becomes higher, beyond certain threshold $\bar{r}$.

Let $\Pi^V(y^*(r);\, r)$ be the lender's expected return when accepting a credit line contract, given an interest rate r. Since either of the two borrowers gets liquidity shock with probability p,

$$\Pi^V(y^*(r);\, r) = 2p \times r \times E[\max\{0, \bar{\rho} - y^*(r)\}] \tag{7}$$

where the first term is the probability with which liquidity shock arises, the second term is the interest rate, and the last term is the amount of liquidity that the lender expects to provide. From (7) and Lemma 1, we get the following lemma.

Lemma 2.

$\Pi^L(y^*(r);\, r)$ *is strictly increasing in r if* $\leq \bar{r}$, *strictly decreasing in r if r* $> \bar{r}$.

This implies that the borrower can promise higher return to the lender by offering higher interest rate only if $r \leq \bar{r}$. When $r > \bar{r}$, however, the borrower cannot promise higher payoff simply by raising the cost of outside liquidity since he should try to hoard more inside liquidity with too costly outside liquidity once the term is contracted. We denote $\Pi^L(\bar{r}) \equiv \bar{\Pi}^L$ as the maximum profit the lender can expect to collect by lending, and the borrower cannot ex ante commit to pay more than this amount to the lender. This is because the choice of y will be made after the credit line contract, and this choice is unobservable and non-verifiable.

Since the lender's rent (outside option value) is Π^V, interest rate r has to satisfy $\Pi^L(y^*(r);\, r) \geq \Pi^V$ in order to induce him to lend which is a participation condition for the lender. Note that only the LHS is a function of r.

Next, we solve for the optimal contract offered by the borrowers given the lender's rent Π^V. Since y is a function of r, the optimal credit line contract boils down to choosing r maximizing the borrower payoff U_i^L which is a function of r, subject to the participation constraint:

$$\max_r U_i^L(y^*(r)) \tag{8}$$

$$\text{s.t. } \Pi^L(y^*(r)) \geq \Pi^V \text{ (IR)} \tag{9}$$

First, consider the case $\Pi^V > \bar{\Pi}^L$. From Lemma 2, we can see that no r can satisfy IR condition (9). The borrower cannot borrow by simply offering higher r in this case since the lender's rent is beyond the level that the borrower can commit to pay. The lender will then choose to be a "buyer" as he can expect higher payoff by going to the secondary market. As we saw before, the optimal response (inside liquidity hoarding) with no interbank lending is $y^* = y^V$ from (4) and the borrower gets $U_i^V(y^V)$.

Now consider the case $\Pi^V \leq \bar{\Pi}^L$. From Lemma 2, $y^* = 0$ if $r < \bar{r}$, and it is obvious from (6) and (7) that $U^L(y^*(r))$ decreases in $\Pi^L(y^*(r))$ when $y^*(r) = 0$. Thus the minimum $\Pi^L(y^*(r))$ maximizes $U^L(y^*(r))$, and IR condition (9) has to bind. From Lemma 2, we can find a unique $r^*(< \bar{r})$ such that (9) binds. The following summarizes our findings.

Proposition 1. (Optimal liquidity provision given the lender's rent)
- *If* $\Pi^V < \bar{\Pi}^L$, *there exists a unique optimal* r^* *such that* $\Pi^L(y^*(r^*)) = \Pi^V$, *and the borrower simply relies on the interbank lending with* $y^* = 0$.
- *If* $\Pi^V \geq \bar{\Pi}^L$, *no lending is provided and the borrower hoard* $y^* = y^V$ *as a cushion.*

This implies that when cost of interbank lending is relatively cheap, the borrowers mainly rely on the lender than hoarding costly liquidity. As the lender's rents become larger, the borrower pays higher interest rate to borrow but still does not hoard own liquidity. But if the rents are beyond $\bar{\Pi}^L$, the borrower cannot borrow from the lender any more. The lender will rather choose to become a "vulture" buyer and the borrowers have to self-prepare by hoarding some liquidity. We should note that the lender's rent Π^V, optimal inside liquidity y^*, interest rate on the loan r^* are all functions of the level of aggregate uncertainty $\bar{\rho}$. This implies that changes in aggregate uncertainty affects the lender's outside option value and ex ante liquidity provision which is the focus of Section 4. where we will argue that the lender's rent becomes larger as aggregate uncertainty increases, affecting ex ante liquidity provision of the borrowers through the higher cost of outside liquidity. In that section, we denote (y^*, r^*, Π^V) of this no risk sharing case as (y_1^*, r_1^*, Π_1^V) to compare them with those in the next case where risks are shared

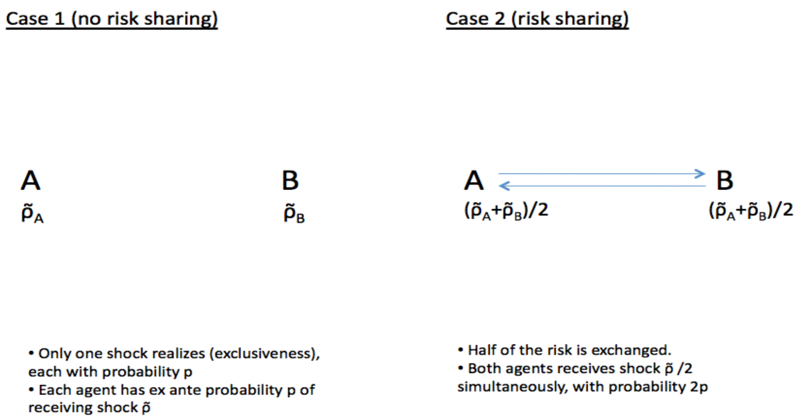

Figure 2. Risk sharing. The two borrowers exchange half of their idiosyncratic risks

among the borrowers.

3.3 Introducing risk sharing

The borrowers may arrange (ex-ante) risk sharing between them.[12] This risk sharing diversifies and smooths their individual interim shock, enabling them to insure each other. Here, I simply characterize risk sharing as an exchange of their idiosyncratic risks; after risk sharing, each borrower owns half of his own risk and half of the other's risk.[13] With the exclusiveness assumption of the two shocks, both borrowers will now be hit by the same shock with probability $2p$, but the size of the shock one gets is half of that without risk sharing, which is distributed uniformly between 0 and $\bar{\rho}/2$. Thus we can denote the individual liquidity shock with risk sharing as

$$\tilde{\rho}_i = \begin{cases} 0 \text{ with prob } 1-2p \\ \dfrac{\tilde{\rho}}{2} \text{ with prob } 2p\text{, where } \dfrac{\tilde{\rho}}{2} \sim U[0, \dfrac{\bar{\rho}}{2}] \end{cases} \tag{10}$$

12) Risk sharing is contracted at $t = -1$ although we will take the risk sharing arrangement as given until Section 4.

13) The merger between the two borrowers can be an alternative interpretation of this risk sharing.

for both borrower $i = A, B$.

This risk sharing is described in Figure 2.

In this context, I again derive the optimal liquidity hoarding y^* and interbank lending cost r^*, along with the lender's outside option value Π^V in the same way as in no risk sharing case. The only difference is that now the borrowers are interconnected (risks are shared) such that distribution of the liquidity shocks they anticipate $(\tilde{\rho}_i)$ will be different.

The borrower's expected payoff is again defined as (3) if the lender refuses to lend (U_i^V), and (6) if he agrees to lend (U_i^L). The difference is now $\tilde{\rho}_i$ follows (10) instead of (1), and $P = \min\{R, M/(2(1-y))\}$ from (2) since the two borrowers are liquidating simultaneously. We can derive y^V, the optimal liquidity hoarding with no interbank liquidity provision, in the same way and now the lender's rent $\Pi^V \equiv \Pi^V(y^V)$ is characterized by

$$\Pi^V(y^V) = 2(1-y^V)\times(R-P)\times\Pr\left(\frac{\tilde{\rho}}{2} > y^V\right)\times 2p$$

$$= 2(1-y^V)\times\left(R-\min\{R, \frac{M}{2(1-y^V)}\}\right)\times\left(1-\frac{2y^V}{\bar{\rho}}\right)\times 2p \tag{11}$$

where the first term is the amount asset the lender expects to buy in the secondary market, the second term is profit per unit of the asset purchased, and the rest is the probability of that event.

If the lender chooses to lend, then the expected payoff at interest rate r is

$$\Pi^L(y^*(r); r) = 2p\times 2r\times E\left[\max\{0, \frac{\tilde{\rho}}{2} - y^*(r)\}\right] \tag{12}$$

which is similar to (7).

Given these, we can solve for the optimal contract and liquidity provision problem by maximizing (8) subject to (9) as before. We first get the similar results as Lemma 1 and 2 in the no risk sharing case.

Lemma 3. *For a given interest rate r,*

- *if $r \le \bar{r}$, then $y^* = 0$. $U_i^L(y(r); r)$ is independent of y, strictly decreasing in r.*

- *if* $r > \bar{r}$, *then* $y^* = \frac{\bar{\rho}}{2}[1 - \frac{(R-1)}{2rp}]$. *where* $\bar{r} = \frac{R-1}{2p}$.

Lemma 4.

$\Pi^L(y^*(r);\ r)$ *is strictly increasing in r if* $r \le \bar{r}$, *strictly decreasing in r if* $r > \bar{r}$.

With these, we can solve for the optimal liquidity hoarding y^* as well as the equilibrium cost of credit r^* as in Proposition 1.

Proposition 2.

- *If* $\Pi^V < \bar{\Pi}^L$, *there exists a unique optimal r* such that* $\Pi^L(y^*(r^*)) = \Pi^V$, and $y^* = 0$.
- *If* $\Pi^V \ge \bar{\Pi}^L$, *the lender refuses to lend, and the borrower hoards* $y^* = y^V$.

The economic interpretations are the same as in the previous case. Note that $(\Pi^V,\ y^*,\ r^*)$ are again functions of $\bar{\rho}$, and we denote them as $(\Pi_2^V,\ y_2^*,\ r_2^*)$ for this risk sharing case. Our next focus is on the sensitivities of these variables with respect to changes in aggregate uncertainty $\bar{\rho}$.

4. COMPARING THE TWO CASES WHEN AGGREGATE UNCERTAINTY FLUCTUATES

4.1 Risk sharing and interbank market fragility

I now discuss the main implication of the model, novel financial fragility emerging from risk sharing. Instead of taking the level of aggregate uncertainty $\bar{\rho}$ as given, we now focus on the sensitivities of interbank lending cost (characterized by Π^V or r^*) and total output y^*, when $\bar{\rho}$ increases. I first show that the lender's rent (equivalently, credit cost) can become more sensitive to aggregate uncertainty fluctuations when risks are shared.

Proposition 3. (Risk sharing exacerbates financial fragility)

- *When* $\bar{\rho}$ *is not very low and market liquidity is not abundant, the risk sharing make the interbank credit cost more volatile to the changes in aggregate uncertainty* $\bar{\rho}$.

- *Formally, there exists $\underline{M}$ and $\bar{M}$ such that if $\underline{M} < M < \bar{M}$,*

$$\frac{d\Pi_2^V}{d\bar{\rho}} > \frac{d\Pi_1^V}{d\bar{\rho}} \tag{13}$$

where subscript 1 represents no risk sharing case and 2 represents risk sharing case.

The assumption $\underline{M} < M < \bar{M}$ implies that cash in the market is large enough to absorb small amount of fire sale without price discount, but is not sufficient to absorb large block trade. The proposition implies that when this is the case, the value of the lender's outside option (wait and buy rather than lend) rises faster as aggregate uncertainty increases (higher $\bar{\rho}$) if risks are shared (Case 2) compared to the case without risk sharing (Case 1). Equivalently, the cost of credit (outside liquidity) rises more rapidly when risks are shared.

The intuition is straightforward from the definition of Π_1^V and Π_2^V in (5) and (11). Note that more number of the borrowers are distressed at the same time when risks are shared, so the fire sale price is lower with more asset liquidation during the distress episodes if it ever happens. This is reflected on the higher profit margin on buying ($R - P$) for Case 2 than that for Case 1. Apparently, this distress event rarely happens with risk sharing since the financial network becomes more resilient to liquidity shock ($\Pr(\tilde{\rho} > y_1^V) > \Pr((\tilde{\rho}/2) > y_2^V)$); there is a trade off between severity and frequency of the distress as discussed in the literature. What's novel here is that small changes in the ex-ante likelihood of financial distress (or tail events) with increased uncertainty, characterized by higher $\bar{\rho}$, can have significantly different effects on the outcome variables in the two cases. In this ex-ante perspective, what matters when considering the effect of the marginal *changes* in aggregate uncertainty is not the size of that event's likelihood itself, but the difference in profit margins since those are the factors that are critical for the sensitivity of the lender's outside option value to fluctuating aggregate uncertainties. With risk sharing, the rent increases more rapidly when the ex-ante likelihood of buying event is increased (with fatter/longer tailed distribution) because of the higher margin on buying whereas it changes only slightly without risk sharing since margin on buying is small (that is, $2(1 - y_2^V\}) \times (R - P_2) >> (1 -$

$y_1^V) \times (R - P_1))$. This brings higher volatility in interbank lending cost and financial fragility with risk sharing. This argument can be summarized as follows.

Corollary 1. (Financial Fragility with Risk Sharing 1)
With risk sharing, the cost of the interbank liquidity rises more rapidly as aggregate uncertainty increases. Credit crunch becomes severe rapidly.

Recall that if the lender's outside option value Π^V is greater than $\overline{\Pi}^L$, outside liquidity evaporates since the lender refuses to lend. If this happens, credit rationing arises, inside liquidity y^V is hoard, and the long term investment drops (Proposition 2, 4).

Now suppose that Proposition 3 holds. As $\bar{\rho}$ increases, Π^V rises faster with risk sharing than without risk sharing. If it eventually hits its upper bound $\overline{\Pi}^L$ with smaller value of $\bar{\rho}$ with risk sharing, this implies that credit rationing and output drops emerges with lower level of aggregate uncertainty when risks are shared.

Corollary 2. (Financial Fragility with Risk Sharing 2)
The following fragility could arise with risk sharing:

1. *Credit is rationed with lower level of aggregate uncertainty.*
2. *Aggregate output suddenly drops with lower level of aggregate uncertainty.*
3. *Both lender and borrowers start to hoard liquid asset with lower aggregate uncertainty.*

In sum, risk sharing can make the lender's outside option value more volatile because of joint failure and corresponding low asset price. This becomes the source of financial fragility. The following example displays these results.

Numerical Example. Consider the following parameters: R = 1.4, M = 1.3, p = 0.2, $\overline{\Pi}^L$ = 0.027, $\bar{\rho} \in [0.65, 0.76]$. The results are summarized in the figures below.

When aggregate uncertainty is low, the borrowers can have access to cheaper credit with risk sharing. This cost of credit (the lender's rent Π^V), however, rises faster with risk sharing as aggregate uncertainty rises (interbank market fragility), and hit $\bar{\rho}$ when $\bar{\rho}$ = 0.75. At this point, credit rationing arises, interbank market breaks

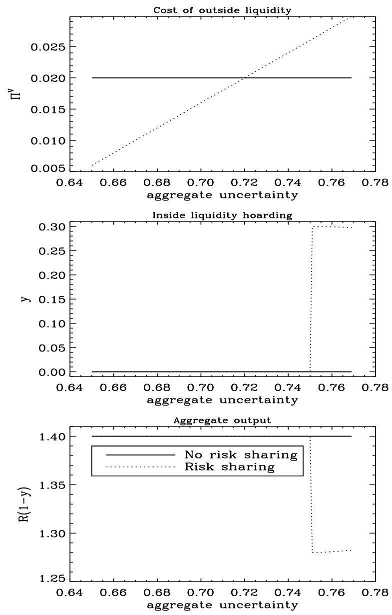

Figure 3. Financial fragilities with risk sharing. Although risk sharing can reduce interbank borrowing cost when aggregate uncertainty is low, both interbank borrowing cost and aggregate output become more sensitive to changes in aggregate uncertainty.

down, and the borrowers cut down their investment level to hoard own liquidity (positive y). Interbank market distress brings a credit crunch in the debt market (lower long term investment), with real effects on the economy through flight to liquidity. Note that these wouldn't have happened if risks had not been shared in our example. Financial fragility emerged through risk sharing in this case.

4.2 Excessive risk sharing

I now discuss that the borrowers may ex ante provide excessive level of risk sharing in seeking cheap credit. I present an example in which the borrowers choose to share their risks even when no risk sharing is socially welfare enhancing.

Note that in this setup, interbank lending cost itself is a mere transfer between the lender and the borrower, and doesn't directly affect aggregate welfare unless credit is rationed (recall that $y^* = 0$

with active interbank market). Thus, aggregate welfare is maximized when the expected total output is maximized with minimal liquidity hoarding y^*. The borrowers, however, try to reduce their cost of credit since it directly affects their payoff. Consider the borrower's ex ante (at $t = -1$) decision of risk sharing before the aggregate uncertainty $\bar{\rho}$ realizes at $t = 0$. If the benefit of cheap credit is greater than the loss from potential credit rationing, the borrowers make a socially suboptimal choice of excessive risk sharing provision. Consider the following 2-state case using the numbers from the previous example.

Suppose that there are two possible states realizing at $t = 0$, H with $\bar{\rho} = \bar{\rho}_H = 0.76$ and L with $\bar{\rho} = \bar{\rho}_L = 0.70$. Let the ex ante probability of state H be 0.05 and that of state L be 0.95 as of $t = -1$. Denote $\Pi^V_{1,H}$ and $\Pi^V_{1,L}$ as the lender's rents for the two state without risk sharing, $\Pi^V_{2,L}$ and $\Pi^V_{2,L}$ with risk sharing.[14)] From the figures of Example 1, we can observe that $\Pi^V_{2,L} < \Pi^V_{1,L} \leq \Pi^V_{1,H} < (\bar{\Pi}^L <)\Pi^V_{2,H}$. This implies that with risk sharing, the borrowers can borrow at a low interest rate in L state, but credit will be rationed in H state whereas no credit rationing arises without risk sharing. However, as H state is unlikely ex ante at $t = -1$, the borrowers choose to share risks in a pursuit of cheap credit in L state. This is not socially optimal since total output is smaller than the first best level in H state with risk sharing, but no credit rationing happens without risk sharing and the total output will always be in its maximum level. Excessive risk sharing arises here bringing less expected total output and higher volatility.

5. POLICY INTERVENTION

As discussed before, cost of credit is a mere transfer between the borrowers and the lender. Hence the policy maker's primary concern is to avoid credit rationing and maximize aggregate output of the economy, rather than reducing the loan cost itself within this setup. First best level of output is produced when $y^* = 0$ (no precautionary saving).

The traditional central bank intervention through open market

14) Note that lower rent implies lower cost of credit, thus higher expected payoff for the borrowers.

operation doesn't directly ease the credit crunch as it doesn't tackle the roots of the high cost of credit: the lender's reluctance to lend comes from their speculation. In this section I analyze three policy interventions which can possibly relax credit rationing: liquidity requirements, asset repurchase, and liquidity injection.

5.1 Liquidity Requirements

The banking industry has argued that strict liquidity requirements are counter-productive since it reduces their long-term investment. In my model, however, liquidity requirements can actually benefit the borrowers by working as a commitment device resolving the time-inconsistency problem.

If the borrower could commit to hoard large amount of liquidity when the lender refuses the credit line offer at $t = 0$, the lender's anticipated profit in the secondary market would be much lower since abundant liquidity should be hoarded. However, this is not a credible threat. The lender knows that the borrowers will not hoard such an excessive liquidity once he turns down the offer (and only hoard the optimal response y^V), thus positive profit as a buyer will do remain.

Now consider mandatory liquidity requirements.[15] If the borrowers are forced to hoard high enough level of own liquidity when outside liquidity drys up, this drives down the lender's rent. Lower rent implies lower cost of outside liquidity, and credit rationing can be avoided. In fact, we can show that

Proposition 4.

For a given level of aggregate uncertainty $\bar{\rho}$, there exists a minimum level of liquidity $\bar{y}$ such that if the borrowers are required to secure at least $\bar{y}$ of liquidity (either own liquidity or lines of credit), the first best level of output without credit rationing is achieved.

Here, social welfare is enhanced without inducing any inefficiency in resource allocation, by changing the outcome in the off-the-equilibrium path. Liquidity requirements act as a commitment device which ex ante rules out financial fragility.

15) Liquidity here also includes committed lines of credit. This comprehensiveness is in line with novel regulations such as LCR and NSFR.

5.2 Asset Repurchase

The government can act as a buyer in the secondary market to stabilize asset price. This directly increases market liquidity and the lender's rents go down, alleviating credit crunch.

This policy can be effective in principle, but in reality the government has the same problem as the other investors (other than the lender) in the secondary market: a lack of special knowledge and potential dislocation costs. The government may not be able to evaluate the complicated assets' value and potentially lose public money by investing in this asset,[16] which imposes political pressure impairing the credibility of the policy actually being implemented. If this is the case, credit crunch won't be alleviated even with the proposed government buy-out program until it gets implemented, as we witnessed during the 2007-09 credit crunch.

5.3 Liquidity injection

Direct liquidity injection can also reduce the lender's rent and cost of credit. I distinguish liquidity injection to the borrower sector and the lender sector, and argue that both policies can be less effective in some cases.

The objective of injecting liquidity to the lender sector is to raise the secondary market asset price P by providing more market liquidity, which reduces the lender's rent and cost of credit. However, there's no guarantee that this policy will work as planned. The lender may use injected liquidity in other uses, in order to keep their rents large enough. If liquidity doesn't flow into the asset market, credit crunch will still remain.

Liquidity injection to troubled borrower can resolve credit crunch, but this causes a moral hazard issue accompanying with bail-out. Knowing that they will be bailed out by the government when in trouble, the borrower tends to take excessive risks. If this liquidity injection is to be implemented in an unanticipated manner, that wouldn't resolve our ex-ante credit crunch since the lender's ex ante expected rent will not be reduced, either.

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16) In other words, the government cannot distinguish between illiquid and insolvent banks.

6. CONCLUSION

This paper presents a model of interbank market stress in which risk sharing exacerbates financial fragility. It has been widely discussed that risk sharing or network formation allows banks to relish diversification benefits and makes the system more robust *ex ante*, but it also interconnects them, and potentially brings systemic distress *ex post*. There is thus a tradeoff between ex ante stability and ex post de-stability. This paper, on the other hand, argues that the system may become more fragile, even in the ex ante perspective.

Financial innovation has provided investors novel ways to diversify their individual risks. This should, in principle, resolve some of the market frictions and enhance social welfare by making the market more complete. The financial market, on the other hand, has become more complicated, and has generated a different kind of friction. When these two are combined, an unanticipated type of fragility could emerge: financial innovation destabilizes the economy.

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A. APPENDIX

A.1 Optimality of credit line contract

I solve for the case 1 (without risk sharing). The proofs for case 2 is similar and omitted.

Claim 1: *The borrowers prefer contracting a line of credit (a lending relationship) at $t = 0$ to borrowing in the spot market at $t = 1$ or not borrowing at all.*

First, notice that the borrower's expected payoffs of no borrowing and spot market borrowing are equivalent. When borrowing in the spot market, the lender tries to extract all the rents which is equal to $(1 - y)(R - P)$. Thus, given inside liquidity y, the borrower's payoff when he borrows from the lender in the spot market at $t = 1$ is

$$(1 - y)R + y - ((1 - y)(R - P)) = ((1 - y)P + y)$$

Since the borrower borrows when $\rho_i > y$, the lender's $t = 0$ objective function is equal to (5), the objective function with no borrowing. Thus, the borrower's choice of y at = 0 is the same in the two cases with identical objective function, which implies they hoard y^V in both cases, and expected payoffs are $U_i^V(y^V)$.

Note that using the definition of Π^V, we can represent it as

$$U_i^V(y^V) = [(1 - y^V)R + y^V] - p\Pi^V - E[\tilde{\rho}_i] \tag{14}$$

Now, we verify that securing a line of credit generates higher expected payoff than (5).

Note that the borrower payoff with credit line is

$$\begin{aligned} U_i^L(y^*, r) &= \max_y[(1 - y)R + y - rE[\max(0, \tilde{\rho}_i - y)] - E(\tilde{\rho}_i)] \\ &= \max_y[(1 - y)R + y - p\Pi^L(r) - E(\tilde{\rho}_i)] \\ &= \max_y[(1 - y)R + y - p\Pi^V - E(\tilde{\rho}_i)] \end{aligned}$$

where the last equality is from binding IR condition (9) such that $\Pi^L(r) = \Pi^V$.

By comparing this and (14), it is obvious that

$$U_i^L(y^*, r) \geq U_i^V(y^V)$$

Thus a credit line contract is weakly better than spot market (or no) borrowing.

Claim 2: *Credit limit of the optimal credit line is no less than $\bar{\rho}$ (full coverage) when credit is not rationed.*

Suppose that a credit line provides a full coverage. When credit is not rationed, $y^* = 0$ as seen in Proposition 2 (a). Therefore, the borrower's payoff in this case is

$$U_i^L = R - \Pi^V - E(\tilde{\rho}_i)$$

since IR condition (9) is binding.

Note that regardless of the amount of credit limit, in expectation Π^V has to be transferred to the lender in order to induce him to lend. It is obvious that U_i^L above is the maximum payoff that the borrower can achieve with any credit limit since no inside liquidity is hoarded. Suppose that full coverage is not provided (credit limit is low) such that the borrower cannot borrow if ρ_i is too large. The cost of outside liquidity is still Π^V, but if the borrower increases the inside liquidity hoarding y, then U_i^L is decreased. Therefore, full coverage with high enough credit limit provides (weakly) higher expected payoff than partial coverage.

A.2 Proofs

Proof of Lemma 1

From the first order condition,

$$(1 - R) + pr - r\frac{y}{\bar{\rho}}p = 0$$

Solving for y,

$$y = \bar{\rho}\left[1 - \frac{(R-1)}{rp}\right].$$

Since $0 \le y \le 1$, from the concavity of U_i^L,

- $y^*(r) = 0$ if $r \le \bar{r}$,
- $y^*(r) = \bar{\rho}[1 - \frac{(R-1)}{rp}]$ if $r > \bar{r}$

where $\bar{r} = (R-1)/p$.

Proof of Lemma 2

From (7),

$$\Pi^L(y^*(r); r) = 2p \times r \times \int_y^{\bar{\rho}} (\rho - y^*(r)) \frac{1}{\bar{\rho}} d\rho$$

From Lemma 1, $y^*(r) = 0$ if $r \le \bar{r}$, so

$$\Pi^L(y^*(r); r) = 2p \times r \times \int_0^{\bar{\rho}} \rho \frac{1}{\bar{\rho}} d\rho$$

which is increasing in r.

Also, $y^*(r) = \bar{\rho}[1 - \frac{(R-1)}{rp}]$ if $r > \bar{r}$, thus

$$\Pi^L(y^*(r); r) = 2p \times r \times \int_y^{\bar{\rho}} (\rho - \bar{\rho}[1 - \frac{(R-1)}{rp}]) \frac{1}{\bar{\rho}} d\rho$$

$$= 2p \times \int_y^{\bar{\rho}} (r\rho - r\bar{\rho} + \frac{(R-1)}{p}) \frac{1}{\bar{\rho}} d\rho$$

Since $\rho < \bar{\rho}$, we get $\frac{\partial \Pi^L}{\partial r} < 0$ in this region.

Proof of Proposition 1

(a) When $\Pi^V < \min\{\hat{\Pi}, \bar{\Pi}^L\}$.

We maximize (6) with respect to r and y such that (9) is satisfied. From Lemma 1, y^* can be represented as a function of r, so our program can be represented by

$$U_i^L(r) = [(1 - y(r))R + y(r)] - p\Pi^L(r) - E[\tilde{\rho}_i]$$
$$\text{s.t. } \Pi^L(r) \geq \Pi^V$$

First, suppose that $r < \bar{r}$. Then from Lemma 1, $y^* = 0$ and our program becomes

$$U_i^L(r) = R - p\Pi^L(r) - E[\tilde{\rho}_i]$$
$$\text{s.t. } \Pi^L(r) \geq \Pi^V$$

It is obvious that $U_i^L(r)$ is maximized when $\Pi^L(r)$ is minimized, therefore $\Pi^L(r) = \Pi^V$. From Lemma 2, $\Pi^L(r)$ is strictly increasing in r, so there exists unique r^* such that $\Pi^L(r^*) = \Pi^V$. The borrower payoff in this case is

$$U_i^L(r^*) = R - p\Pi^V - E[\tilde{\rho}_i] \tag{15}$$

Now suppose $r > \bar{r}$, then (6) becomes

$$U_i^L(r) = [(1 - y^*(r))R + y^*(r)] - p\Pi^L(r) - E[\tilde{\rho}_i]$$

Since $y^*(r) > 0$ and $\Pi^L(r) \geq \Pi^V$, this is strictly less than (15), thus $y^* = 0$ and r^* that we derived above are the optimal solution.

(b) When $\Pi^V \geq \min\{\hat{\Pi}, \bar{\Pi}^L\}$.

The lender refuses to contract a line of credit since his expected payoff is higher when he chooses to buy in the secondary market. Thus, $y^* = y^V$ which given by (4).

Proof of Proposition 3

Note that from (5)

$$\Pi_1^V = p(1 - y_1^V) \times (R - P_1\}) \times (1 - \frac{y_1^V}{\bar{\rho}})$$
$$= p(1 - y_1^V) \times (R - \min\{R, \frac{M}{1 - y_1^V}\}) \times (1 - \frac{y_1^V}{\bar{\rho}})$$

and from (10)

$$\Pi_2^V = 2p(1 - y_2^V) \times (R - P_2\}) \times (1 - \frac{2y_2^V}{\bar{\rho}})$$
$$= 2p(1 - y_2^V) \times (R - \min\{R, \frac{M}{2(1 - y_2^V)}\}) \times (1 - \frac{2y_2^V}{\bar{\rho}})$$

We can derive the exact condition by plug in y_1^V, y_2^V and differentiate with respect to $\bar{\rho}$, but the closed form solutions are very complicated. Not very small $\bar{\rho}$ is needed so that the insiders don't fully insure in case 2. Suppose that M is chosen such that $R - P_1 = R - \min\{R, M/(1 - y_2^V)\}$ is small but $R - P_2 = R - \min\{R, M/(2(1 - y_2^V))\}$ is not very small. For this M, as long as $R - P_2 >> R - P_1$ then $\partial\Pi_2^V/\partial\rho > \partial\Pi_1^V/\partial\rho$ holds from the previous equations. By continuity, there exist $\underline{M}$ and $\bar{M}$ such that the lender's rent becomes more sensitive with risk sharing.

Proof of Proposition 4

Note that from (5) and (10), Π^V is a function of $\bar{\rho}$ and y. Observe that Π_s^V is increasing in $\bar{\rho}$ and decreasing in y. Therefore for any $\bar{\rho}$, we can find $\bar{y}$ such that

$$\Pi^V(\bar{y}; \bar{\rho}) \leq \min\{\hat{\Pi}, \bar{\Pi}^L\}$$

With liquidity requirements of $\bar{y}$, the lender's expected profit in the secondary market is less than $\min\{\hat{\Pi}, \bar{\Pi}^L\}$, which means that cost of interbank liquidity is less than the upper bound beyond which credit is rationed. The lender then provides liquidity since he wouldn't get higher payoff by rejecting the credit line offer.

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