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Heterogeneous Expectations, Asset Prices, and Trading Volume under a Non-expected Utility Function with CARA*

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ABSTRACT

Using a non-expected utility function that exhibits constant relative risk aversion (CRRA), Cho (2001) explores a theoretical model of asset pricing under heterogeneous beliefs in the case where only one risky asset is traded. This paper extends his work into the case where agents trade a risky asset and the riskless asset as well, adopting a non-expected utility function that exhibits constant absolute risk aversion (CARA). In a variant of the general equilibrium setting of Lucas (1978), major findings of the paper are as follows: (i) When agents differ only in expectations about future dividends, the question of who is the buyer and who is the seller of each asset depends solely on the degree of optimism. Unlike the case of Cho (2001), there is no role of intertemporal substitution. (ii) Increased dispersion of expectations will raise the risk-free rate and lower the risky asset's price. This result is consistent with that of Abel (1990). (iii) Although the equity premium goes up as a consequence of result (ii), heterogeneity per se does not help to resolve the puzzle posed by Mehra & Prescott (1985) and Weil (1989). (iv) The trading volume of the risky asset increases proportionately with the cross-sectional variance of expectations, and the same is true for the riskless asset. (iv) An increase in the risk-free interest rate will reduce the trading volume of the riskless asset unless the intertemporal substitution parameter is less than 1/2. In addition to these findings, many more comparative statics results are obtained from closed-form solutions for asset prices and trading volume.

Keywords: Heterogeneous Expectations; Non-expected Utility Function; Constant Absolute Risk Aversion; Asset Prices; Trading Volume

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I. INTRODUCTION

While the assumption of homogeneous expectations about future asset returns plays an important role in most of well-known asset pricing models,[1)] numerous studies relax this assumption and examine the effects of heterogeneity among economic agents in a variety of frameworks. This study presents a theoretical model to investigate the issue in a general equilibrium setting, using a non-expected utility function that exhibits constant absolute risk aversion (CARA). Specifically, assuming heterogeneous beliefs about the mean of future dividends, the model in this paper will derive closed-form solutions for a risky asset's price, the risk-free rate, the equity premium, the trading volume of the risky asset, and the amount of riskless lending or borrowing as functions of economic parameters. These solutions make it possible to perform various comparative static analyses and to compare the results with those found in the existing literature.

In line with the work done by Cho (1992, 2001), this paper is especially concerned with the issues addressed in earlier studies such as Miller (1977), Varian (1985, 1987), and Abel (1990). Miller asserts that in a market with short sale constraints, the demand for a risky security will primarily come from the minority who hold the most optimistic expectations about security payoffs. Thus, the larger the dispersion of opinions concerning the asset's return (with the cross-sectional mean of the opinions held constant), the higher the market clearing price will be. His arguments seem to be plausible intuitively, but his model is far from being formal.[2)]

Contrary to Miller's result, Varian (1985) shows that, unless risk aversion declines too rapidly, increased dispersion of beliefs will generally be associated with reduced asset prices in a complete Arrow-Debreu securities market. Although he develops a measure of the degree of dispersion elegantly, his model has a serious

1) For example, see the standard CAPM by Sharpe, Lintner, and Mossin, the Intertemporal CAPM by Merton, the Consumption CAPM by Breeden, the representative consumer model by Lucas (1978), etc.

2) Sheikman and Xiong (2004) formally derive the same result in a model assuming that agents' expectations are uniformly distributed. However, the effect of heterogeneity in their model is confounded with that of the number of agents. Moreover, the risk-free rate is exogenously given instead of being determined endogenously.

problem in that the belief parameter is specified as a mixture of an exogenous variable (subjective probability measure) and an endogenous variable (Lagrangian multiplier).

Cho (1992) analyzes the issue in a two-person general equilibrium setting of Lucas (1978) using a time-additive expected utility with constant relative risk aversion (CRRA). He shows that in a market where only one risky stock is traded, increased dispersion of beliefs will lower the asset price unless the CRRA coefficient less than one. Cho (2001) extends his previous work adopting the two-period version of the non-expected utility function developed by Epstein and Zin (1988) [EZ utility, henceforth]. Main findings there are: (i) Whether the optimistic person becomes the seller or the buyer of the asset depends solely on the degree of intertemporal substitution. (ii) Increased dispersion of beliefs may either reduce or raise the asset price depending on the interaction between the roles played by aversion to intertemporal substitution and risk aversion. Cho (1992) and Cho (2001) also find that the trading volume of the risky asset increases with increased dispersion of beliefs.

Abel (1990) considers the case where agents trade a risky stock and the riskless asset as well. In his model, the risk-free rate is determined endogenously. Using a time-additive expected utility that exhibits constant absolute risk aversion (CARA), he shows that increased cross-sectional variance of subjective expectations reduces the risky asset's price, raises both the risk-free rate and the equity premium. His model does not address the effect of heterogeneity on the trading volume, though.

The model presented below may be regarded as an extension of Cho (2001) on one hand, and that of Abel (1990) on the other. It extends Cho (2001) by allowing the riskless asset to be traded in the capital market. It extends Abel (1990) by replacing time-additive expected exponential utility with non-expected utility preferences which are iso-elastic intertemporally, but exponential in risk dimension. It is well-known that the time-additive expected utility function used by Abel cannot distinguish between aversion to intertemporal substitution and risk aversion. However, along with EZ utility that Cho (2001) used, the non-expected utility function adopted in this paper makes a clear distinction between the two disparate preference components. The difference between the two utility functions is that while the former exhibits CRRA, the latter exhibits CARA.

Using the latter utility function has several advantages for the purpose of this paper. First, it is well suited to the case of the existence of the riskless asset by allowing the problem solvable. By comparison, adopting EZ utility in this case does not make the model tractable. Second, it enables one to see the role played by intertemporal substitution in analyzing the effect of heterogeneity on asset pricing, which time-additive expected utility does not. Third, along with the normality assumption about asset payoffs, it produces fruitful comparative statics results as well as closed-form solutions for economic variables.

More recently, much more research has been done on asset pricing under heterogeneous beliefs for diverse purposes. Detemple and Murphy (1994) examine the effects of different prior beliefs on asset prices and holdings in continuous time production economy with agents having log utility functions. In a similar setting, Zapatero (1998) investigates the effects of financial innovation on the volatility of interest rates in a pure exchange economy. Basak (1999), in continuous time also, shows how non-fundamental risk affects asset prices when agents have heterogeneous beliefs about a source of the risk. Li (2007) introduces a continuous-time model that provides a closed-form solution of the stock price in the Lucas (1978) economy with agents having different beliefs and log utility. Notably, his model characterizes the volatility of stock price, which none of the above-mentioned papers does.

Jouini and Napp (2006, 2007) take somewhat unique approach. They construct a consensus agent who, when she/he has the whole endowment of the economy, generates the same equilibrium price as in the original economy with heterogeneous agents. The belief of this consensus agent is simply an average of individual beliefs, but her/his discount factor turns out to be proportional to the dispersion of beliefs and might be positive or negative depending on whether the agent is 'cautious' or not in the context of HARA(hyperbolic absolute risk aversion) class utility functions.[3] In the sense that the discount factor usually reflects the degree of risk, they interpreted the dispersion of beliefs as a source of risk. One related work is Bhamra and Uppal (2014). Using the 'catching up with the Joneses' utility function, and assuming that agents differ in beliefs, time

3) In HARA class utility functions of the form $-\frac{u'}{u''} = A + BW$, coefficient B represents cautiousness.

preferences (utility discount factor), risk aversion, and consumption habit, they provide closed-form solutions for the risk-free interest rate, the stock price, equity premium, and volatility of stock returns, the term structure of interest rates.

Meanwhile, Sheinkman and Xiong (2003) present a continuous-time model in infinite horizon with a single risky asset with limited supply and many risk-neutral agents. In their model, disagreements among agents are generated by overconfidence and this causes a significant bubble component in the asset price in an environment with short-sale restrictions. Chiarella and He (2002) consider the case where agents have different degrees of risk aversion and adapt their beliefs over time. They incorporate risk and learning schemes into a discounted present value model and find that the external noise and leaning schemes can significantly affect the asset price dynamics.

In addition, Chiarella, Dieci, and He (2007) develop a model in which agents whose beliefs differ in both first and second moment of asset returns trade multiple risky assets and a riskless asset. Hansen (2015) also considers a case where heterogeneous agents trade many assets, taking the Monte-Carlo simulation approach. A numerical analysis generates a low level of risk-free rate, reasonable equity premia and return volatilities. Huang, Qiu, Shang, and Tang (2013) present a model in which agents with heterogeneous beliefs care about relative investment performance. They find that concern about relative performance leads agents to trade more similarly, and that similar trading decreases volatility, and the impact of dominant agents. Baker, Hollifield, and Osambela (2016) show that, in a production economy with recursive preferences, disagreement among investors generates dynamic aggregate investment leading to stochastic volatility in aggregate consumption, investment, and equity return.

As far as the effects of heterogeneous beliefs on trading volume are concerned, related literature mostly agrees that increased dispersion of beliefs will cause increased trading. While earlier studies such as Karpoff (1986) and Varian (1987) draw this conclusion analytically, Kim (1983) and Ajinkya, Atiase, and Gift (1991) provide empirical evidence on this relationship. Using the experimental method, Dinh and Gajewski (2015) also show that the dispersion of beliefs is the main driver behind trading volume, and that their relationship is concave. One exception to this line is the study by Pfleiderer (1984), which reports that expected volume of trade is a decreasing function of the dispersion of expectations.

Assuming that agents receive common information but differ in the way of interpreting this in formation, Harris and Raviv (1993) derive the results that absolute price changes and volume are positively correlated, consecutive price changes are negatively autocorrelated, volume is positively autocorrelated. Wang (1994) also find that trading volume is positively correlated with absolute changes in prices and dividends.

All these studies analyze trading volume for risky assets, not for the riskless asset. By comparison, the model in this paper makes it possible to derive trading volume for both assets as linear functions of the cross-sectional variance of beliefs. To the author's knowledge, no other paper has shown this result yet.

The remainder of this paper is organized as follows. In section II, a variant of the representative consumer economy of Lucas (1978) is described, and the demand for the risky asset in its equilibrium is derived. Section III analyzes the effects of heterogeneous beliefs on asset prices based on closed-form solutions for the risky asset's price, the risk-free rate, and the equity premium. In addition, it provides various comparative statics results. Section IV examines the effects of heterogeneous beliefs on trading volume for the risky and the riskless assets. Section V summarizes and concludes.

II. THE MODEL

Consider a pure exchange economy of Lucas (1978) type in which non-storable consumption goods are produced by one productive unit, the ownership of which is represented by a divisible share of a risky stock. Suppose that there are n economic agents (i = 1, 2, …, n) who consume goods at two points of time (t = 0, 1), and with identical preferences. At time 0, each consumer is initially endowed with a fraction (z^i) of the share, entitling it to the current dividend (y_0) proportionately. Given the dividend income ($z^i \cdot y_0$) at time 0, each agent allocates it between current consumption (c_0) and savings for future consumption (c_1). Savings are made by investments in the risky asset and the risk-free asset as well. The (gross) risk-free rate is denoted by R_f.

At the time of consumption-savings decision (t = 0), the amount of future aggregate dividend (y_1) is unknown (random variable). Suppose that each agent is forming expectations about y_1 as follows:

$$y_1 \sim N(\mu^i, v) \tag{1}$$

where N stands for 'normally distributed', μ and v denote the mean and the variance of the future dividend, respectively. Notice that agents have common beliefs about the volatility (variance) but have different beliefs about the mean.[4] In this model, different beliefs about the mean are referred to as heterogeneous expectations.[5]

Forming expectations about the future dividend, agents make portfolio decisions that are concerning how much to invest in the risky and the riskless assets, respectively. As a result, the current and future consumptions of agent i are determined, respectively, as follows:

$$c_0^i = z^i \cdot y_0 - p \cdot q^i - l^i, \quad c_0^i \geq 0 \tag{2}$$

$$c_1^i = (z^i + q^i) \cdot y_1 + l^i \cdot R_f, \quad c_0^i \geq 0 \tag{3}$$

where p (>0) is the ex-dividend price of the risky asset, which is competitively determined in the stock market and is denominated in terms of units of consumption good, q^i is the amount of the share to be bought ($q^i > 0$) or sold ($q^i < 0$) by agent i, and l^i is her/his riskless lending ($l^i > 0$) or borrowing ($l^i < 0$). Note that since y_1 is a random variable, c_1^i is also a random variable.

The objective of agent i is to maximize the utility function of the following form:

$$U^i = \left[c_0^{i\,1-\rho} + \beta \cdot \hat{c}_1^{i\,1-\rho} \right]^{\frac{1}{1-\rho}}, \quad \text{where } c_1^i \equiv -\frac{1}{\theta} \cdot \ln E^i \left(e^{-\theta \cdot c_1^i} \right) \tag{4}$$

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4) This assumption is based on the common argument that mean returns are much harder to estimate than return volatilities. See Williams (1977) and Merton (1980)

5) In this paper, the reason for disagreement among investors is not made explicit. Sheinkman and Xiong (2003) claim that overconfidence, the belief of an agent that her/his information is more accurate that it is, may be an important source of disagreement. Varian (1987), and Harris & Raviv (1993) claim that divergence of opinions may be caused by the difference in the way of interpreting information.

In equation (4), U is utility, ρ is the parameter governing intertemporal substitution $[0 < \rho \neq 1]$, θ is the coefficient of absolute risk aversion $[\theta > 0]$, E^i is the (subjective) expectation of agent i conditional on the available information at time 0, β is the utility discount factor. The above objective function is a representation of preferences that obey the hypothesis of Kreps and Porteus (1978), the form of which is similar to the two-period version of EZ utility.[6] Note that function U takes a CES form and is an aggregator of current consumption, c_0, and the certainty equivalent of future consumption, $\hat{c}_1$.[7] As far as attitudes toward risk are concerned, the utility function in equation (4) exhibits constant absolute risk aversion (CARA) while EZ utility exhibits constant relative risk aversion (CRRA).

One notable characteristic of the above utility function is that it makes distinction between an agent's attitudes toward time variation and riskiness in her/his consumption profile. In the traditional time-additive expected utility framework, these two preference components, referred to as respectively as aversion to intertemporal substitution and risk aversion, are mixed up so that it is impossible to tell one from the other. Like the non-expected utility function of Epstein and Zin (1989), the utility function in (4) enables one to clarify the respective roles played by these preference components in analyzing many economic problems.

Demand for the Risky Asset

The first-order conditions for maximizing the utility in equation (4) subject to equations (2) and (3) can be derived as follows.

$$(c_0^i)^{-\rho} \cdot p = \beta \cdot (\hat{c}_1^i)^{-\rho} \cdot \frac{1}{E^i(e^{-\theta \cdot c_1})} \cdot E^i(e^{-\theta \cdot c_1} \cdot y_1) \tag{5}$$

$$(c_0^i)^{-\rho} = \beta \cdot (\hat{c}_1^i)^{-\rho} \cdot \frac{1}{E^i(e^{-\theta \cdot c_1})} \cdot E^i(e^{-\theta \cdot c_1} \cdot R_f) \tag{6}$$

The last term of the right-hand side of equation (5) is divided into

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6) Weil (1993) used the utility function in (4) to study an optimal consumption-savings problem for infinitely-lived agents.

7) Throughout this paper, the hat (ˆ) notation is used to indicate the certainty equivalent of any random variable and is computed in the same manner as in equation (4).

two parts as below.

$$E^i(e^{-\theta \cdot c_1} \cdot y_1) = E^i(e^{-\theta \cdot c_1}) \cdot \mu^i(y_1) + Cov^i(e^{-\theta \cdot c_1}, y_1) \tag{7}$$

where $Cov(\cdot,\cdot)$ denotes the covariance. By using equations (1) and (3), and by Stein's lemma, it is possible to calculate the covariance term in equation (7) as follows

$$\begin{aligned} Cov^i(e^{-\theta \cdot c_1}, y_1) &= -\theta \cdot E^i(e^{-\theta \cdot c_1}) \cdot Cov^i(c_1, y_1) \\ &= -\theta \cdot (z^i + q^i) \cdot E^i(e^{-\theta \cdot c_1}) \cdot Cov^i(y_1, y_1) \\ &= -\theta \cdot (z^i + q^i) \cdot E^i(e^{-\theta \cdot c_1}) \cdot v \end{aligned} \tag{8}$$

Meanwhile, equation (6) can be rearranged as follows:

$$R_f = \left(\frac{1}{\beta}\right) \cdot (c_0^i)^{-\rho} \cdot \left(\hat{c}_1^i\right)^{\rho} \tag{9}$$

Substituting equations (8) and (9) into equation (5) gives the following relationship.

$$\begin{aligned} p &= \beta \cdot (c_0^i)^{\rho} \cdot (\hat{c}_1^i)^{-\rho} \cdot [\mu^i - \theta \cdot (z^i + q^i) \cdot v] \\ &= \frac{1}{R_f} \cdot [\mu^i - \theta \cdot (z^i + q^i) \cdot v] \end{aligned} \tag{10}$$

Rearranging equation (10) will give the demand for the risky asset of investor i, that is, the amount of share of the risky asset that investor i will trade.

$$q^i = \frac{\mu^i - p \cdot R_f}{\theta \cdot v} - z^i \tag{11}$$

A couple of comments deserve to be made about equation (11). First, it is possible for q^i to be negative, which implies that short selling is allowed in this economy. Investors who are sufficiently pessimistic (that is, μ^i is small enough) or relatively more endowed (that is, z^i is large enough) may sell the risky asset short. Second, the demand function is independent of the intertemporal substitution parameter. The latter result is in the same vein with the finding of Svensson (1989) that the optimal portfolio decision is governed by risk aversion but not by intertemporal substitution.

Equilibrium

For the economy to be in equilibrium, the following market clearing conditions should be met.

$$\sum_{i=1}^{n} c_0^i = y_0 \tag{12}$$

$$\sum_{i=1}^{n} c_1^i = y_1 \tag{13}$$

These equations will hold if the following conditions are satisfied.

$$\sum_{i=1}^{n} z^i = 1 \tag{14}$$

$$\sum_{i=1}^{n} q^i = 0 \tag{15}$$

$$\sum_{i=1}^{n} l^i = 0 \tag{16}$$

Summing equation (11) over i and using conditions (14) and (15) yields the following relationship.

$$\sum_{i=1}^{n} q^i = \frac{\sum_{i=1}^{n} \mu^i - n \cdot p \cdot R_f}{\theta \cdot v} - 1 = 0 \tag{17}$$

Rearranging equation (17) will give

$$\sum_{i=1}^{n} \mu^i - n \cdot p \cdot R_f = \theta \cdot v \tag{18}$$

Dividing both sides of equation (18) by n and rearranging the resulting equation with respect to the risky asset's price p gives

$$p \cdot R_f = \bar{\mu} - \left(\frac{1}{n}\right) \cdot \theta \cdot v \tag{19}$$

where $\bar{\mu}[\equiv (1/n)\sum_{i=1}^{n}\mu^{i}]$ is the cross-sectional average of μ^i. The right-hand side of equation (19) can be interpreted as the risk-adjusted mean value of the future dividend. So the risky asset's price turns out to be its present value discounted by the risk-free rate. Note that in order for the risky asset's price and the risk-free rate to be positive, the following condition should be met: $\bar{\mu} > (1/n)\theta v$.

Substituting equation (19) into equation (11) gives agent i's holdings of the risky asset after trades.

$$z^{i} + q^{i} = \frac{\mu^{i} - \bar{\mu}}{\theta \cdot v} + \frac{1}{n} \tag{20}$$

Equation (20) indicates that if all investors have homogeneous expectations $(\mu^i = \mu, \forall i)$, the first term on the right-hand side will vanish and thus their holdings of the risky asset will be the same $(1/n)$ regardless of their endowments. This result reflects the well-known property that agents with CARA utility will invest the same amount of money in the risky asset without regard to the level of their wealth. Therefore, different holdings of the risky asset among investors are caused solely by differences in beliefs about the mean payoffs.

III. THE EFFECTS OF HETEROGENEOUS EXPECTATIONS ON ASSET PRICES

The Risk-free Rate and the Risky Asset's Price

If the risk-free interest rate R_f in equation (19) were exogenously given, the heterogeneity of expectations would not affect the risky asset's price. However, R_f is endogenously determined in this model. Let us turn to derive the risk-free rate as a function of exogenous parameters using equation (9). First of all, the certainty equivalent of future consumption of agent i is calculated as below.

$$\begin{aligned}\hat{c}_1^{i} &\equiv -\frac{1}{\theta}\cdot \ln E^{i}(e^{-\theta \cdot c_1}) \\ &= -\frac{1}{\theta}\cdot \ln\left[\exp\left(-\theta \cdot E^{i}(c_1^{i}) + (1/2)\cdot \theta^{2}\cdot Var(c_1^{i})\right)\right]\end{aligned}$$

$$= E^i(c_1^i) - \left(\frac{1}{2}\right) \cdot \theta \cdot Var(c_1^i)$$

$$= \left(z^i + q^i\right) \cdot \mu^i + l^i \cdot R_f - \left(\frac{1}{2}\right) \cdot \theta \cdot \left(z^i + q^i\right)^2 \cdot \upsilon$$

$$= \left(\frac{\mu^i - \bar{\mu}}{\theta \cdot \upsilon} + \frac{1}{n}\right) \cdot \mu^i + l^i \cdot R_f - \left(\frac{1}{2}\right) \cdot \theta \cdot \upsilon \cdot \left(\frac{(\mu^i - \bar{\mu})^2}{(\theta \cdot \upsilon)^2} + 2 \cdot \frac{\mu^i - \bar{\mu}}{\theta \cdot \upsilon} \cdot \frac{1}{n} + \frac{1}{n^2}\right)$$

$$= \frac{(\mu^i)^2 - \bar{\mu}^2}{2 \cdot \theta \cdot \upsilon} + \frac{1}{n} \bar{\mu} + l^i \cdot R_f - \frac{\theta \cdot \upsilon}{2 \cdot n^2} \tag{21}$$

Summing $\hat{c}_1^i$ over i and simplifying will give

$$\sum_{i=1}^{n} \hat{c}_1^i = \frac{\sum_{i=1}^{n} (\mu^i)^2 - n \cdot \bar{\mu}^2}{2 \cdot \theta \cdot \upsilon} + \bar{\mu} - \frac{\theta \cdot \upsilon}{2} \cdot \sum_{i=1}^{n} \frac{1}{n^2} \tag{22}$$

Dividing both sides of equation (22) by n will give the average certainty equivalent of future consumption:

$$\bar{\hat{c}}_1 \equiv \frac{1}{n} \cdot \sum_{i=1}^{n} \hat{c}_1^i = \frac{Var^c(\mu^i)}{2 \cdot \theta \cdot \upsilon} + \frac{1}{n} \cdot \bar{\mu} - \frac{\theta \cdot \upsilon}{2} \cdot \frac{1}{n^2} \tag{23}$$

In equation (23) $Var^c(\mu^i)$ denotes the cross-sectional variance of subjective expectations, which measures the degree of dispersion in beliefs.

Meanwhile, equation (9) can be rewritten with respect to the current consumption of agent i:

$$c_0^i = (\beta \cdot R_f)^{-\frac{1}{\rho}} \cdot \hat{c}_1^i \tag{24}$$

Summing c_0^i over i and taking the average will give

$$\bar{c}_0 \equiv \frac{1}{n} \cdot \sum_{i=1}^{n} c_0^i = \frac{1}{n} \cdot y_0 = (\beta \cdot R_f)^{-\frac{1}{\rho}} \cdot \bar{\hat{c}}_1 \tag{25}$$

Rearranging equation (25) with respect to R_f and replacing $\bar{\hat{c}}_1$ with equation (23) will produce one of the main results of this paper:

$$R_f = \frac{1}{\beta} \cdot y_0^{-\rho} \cdot \left(\frac{n \cdot Var^c(\mu^i)}{2 \cdot \theta \cdot \upsilon} + \bar{\mu} - \frac{\theta \cdot \upsilon}{2} \cdot \frac{1}{n}\right)^{\rho} \tag{26}$$

Observing equation (26) makes plenty of comparative static analyses possible. First of all, an increase in the cross-sectional variance of subjective expectations will raise the risk-free interest rate. And the effect is stronger with (i) the higher degree of intertemporal substitution (ρ); (ii) the lower degrees of risk (υ) and risk aversion (θ); (iii) the lower value of discount factor (β); (iv) the larger certainty equivalent of aggregate consumption growth[8]; (v) the larger size of population (n). Assuming no disagreement among agents [$Var^c(\mu^i) = 0$], the risk-free rate itself will increase under conditions (i) to (v). These results are consistent with those of earlier studies on this subject [see Kimball (1990), Barsky (1989), Abel (1988), Epstein (1988)].[9]

Substituting equation (26) into equation (19) will give the risky asset's price as a function of state variables of the economy.

$$p = \left(\bar{\mu} - \frac{1}{n} \cdot \theta \cdot \upsilon\right) \cdot \beta \cdot y_0^{\rho} \cdot \left(\frac{n \cdot Var^c(\mu^i)}{2 \cdot \theta \cdot \upsilon} + \bar{\mu} - \frac{\theta \cdot \upsilon}{2} \cdot \frac{1}{n}\right)^{-\rho} \tag{27}$$

Equation of (27) is another key result of this paper. An increase in the cross-sectional variance of subjective expectations will lower the risky asset's price. That is, the asset price is negatively related with the degree of disagreement among agents. Taking the log on both sides of equation (27) shows that the negative effect is stronger with (i) the higher degree of intertemporal substitution (ρ); (ii) the lower degrees of risk (υ) and risk aversion (θ); (iii) the larger population (n).

Abel (1990) derives a similar formula for the risky asset's price in a different setting, but there are no roles of intertemporal substitution and population size in his model. Cho (2001) considers a case in which heterogeneous agents with EZ utility trade only one risky asset. He finds that the effect of heterogeneous beliefs depends critically on the interaction between risk aversion and intertemporal substitution. For example, an increased dispersion of beliefs will increase (decrease) the risky asset's price if the intertemporal

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8) $\left(\bar{\mu} - \frac{\theta \cdot \upsilon}{2} \cdot \frac{1}{n}\right) / y_0$ represents the certainty equivalent of consumption growth.

9) Result (ii) is consistent with the finding of Kimball (1990) that increasing uncertainty reduces the risk-free rate, which is called 'precautionary saving effect.' This effect obtains if preferences exhibit positive 'prudence', which is measured by $-u'''/u''$. The utility function in equation (4) implies constant prudence.

substitution parameter is 0.5 (2.0) and the relative risk aversion coefficient is 2. Contrasted with this result, equation (27) shows that no such role is played by the interaction between the two disparate preference components.

With homogeneous expectations in equation (27) [$Var^c(\mu^i) = 0$], it is also possible to verify that the risky asset's price is increasing with the current dividend (y_0) and the discount factor (β). It is also increasing with the mean of future dividend if $\rho \leq 1$. Moreover, it can be shown that the price is decreasing with the risk (υ) and the degree of risk aversion (θ), and increasing with the population size (n) if $\rho \leq 2$.[10] The last result is quite interesting, the reason being that the dispersion of aggregate economic uncertainty among a larger number of agents will effectively reduce the risk taken by an individual agent. The above results are mostly consistent with those findings of Abel (1988), Epstein (1988), and Barsky (1989).

One notable feature of the asset price in equation (27) is that it satisfies the aggregation property in the sense of Rubinstein (1974). An economy with heterogeneous agents satisfies the aggregation property if the equilibrium prices are determined independent of the distribution of initial endowments. It is clearly seen that the price in equation (27) does not depend on the parameter z that denotes endowments.[11]

The Equity Premium and the Puzzle by Mehra and Prescott

The puzzle posed originally by Mehra and Prescott (1985)[12] on the asset pricing model of Lucas (1978) consists of two parts: When asset returns predicted by the model are compared to their historical averages, the equity premium is too small and the risk-free rate is too large. These phenomena are referred to as the equity premium puzzle ('puzzle I', henceforth) and the risk-free rate puzzle ('puzzle II', henceforth), respectively. To resolve these puzzles, one needs a

10) To check the signs of these effects, the condition $\bar{\mu} > (1/n)\theta\upsilon$ is needed. When the non-expected utility function with constant relative risk aversion is used, the sign of the effect varies depending on whether ρ is less or greater than one.

11) The sufficient condition for the aggregation property to hold under heterogeneous beliefs is that agents have the generalized negative exponential utility class. In this case, differences among agents in the risk aversion coefficient and in the time preference (discount factor) are allowed.

12) Also see Weil (1989).

framework that raises the equity premium and lowers the risk-free rate by a big margin. Kim and Cho (2019) recently demonstrates that the utility function in equation (4) has a good potential to alleviate both puzzles significantly if ρ is sufficiently small. In this section, I explore whether heterogeneity of expectations per se can further improve their results in addition to the usefulness of the non-expected utility in equation (4).

For every realization of the future dividend, the ex-post equity premium in the model of this paper can be written as follows:

$$\pi \equiv \frac{y_1}{p} - R_f = \left(\frac{y_1}{p \cdot R_f} - 1 \right) \cdot R_f = \left(\frac{y_1}{\bar{\mu} - \frac{1}{n} \cdot \theta \cdot v} - 1 \right) \cdot R_f \tag{28}$$

Suppose that the consensus belief about the mean of future dividend is the same as the average value of realized future dividends. Then the average value of the observed ex post equity premium can be written as

$$\bar{\pi} = \left(\frac{\bar{\mu}}{\bar{\mu} - \frac{1}{n} \cdot \theta \cdot v} - 1 \right) \cdot R_f \tag{29}$$

Since the risk-free rate in equation (29) is endogenously determined as in equation (26), substituting equation (26) into equation (29) will show that the equity premium is positively related to the cross-sectional variance of subjective expectations. Thus one might conclude that the existence of heterogeneity can be of help in resolving the equity premium puzzle.

However, this conclusion holds true for ‘puzzle I’ only. To the extent that the risk-free rate is positively related to the dispersion of subjective expectations, the conclusion is incorrect as far as ‘puzzle II’ is concerned. Therefore, heterogeneity of expectations per se cannot resolve the equity premium in the sense that it worsens ‘puzzle’ II although it may substantially alleviate ‘puzzle I’.

IV. THE EFFECTS OF HETEROGENEOUS EXPECTATIONS ON TRADING VOLUME

According to Grossman and Stiglitz (1980), trade among individuals occurs either because tastes (risk aversion) differ, endowments differ, or beliefs differ. Since the model presented so far assumes that agents differ in beliefs and endowments, it is natural that trades are generated in the preceding model. Moreover, the claim of Grossman and Stiglitz implies an intuitive conjecture that the more different agents are, the more they trade. Let us now investigate the validity of this conjecture through formal analyses.

Trading Volume of the Risky Asset

To measure how much agents trade the risky asset, one needs to aggregate its demands of all agents ($i = 1, 2, \ldots, n$). Rewriting equation (20) will give the demand for the risky asset of agent i.

$$q^i = \frac{\mu^i - \bar{\mu}}{\theta \cdot v} + \frac{1}{n} - z^i \tag{30}$$

By observing equation (30), one can determine who becomes the buyer and who becomes the seller of the risky asset. If agent i is relatively optimistic ($\mu^i > \bar{\mu}$) and less endowed than the average ($z^i < 1/n$), she/he becomes the buyer. If agent i is relatively pessimistic ($\mu^i > \bar{\mu}$) and more endowed than the average ($z^i < 1/n$), she/he becomes the seller. Otherwise, it is not possible to tell because the answer depends on the interaction between the degree of optimism and the amount of endowment.

These results are contrasted with the result of Cho (2001), which considers a case where only one risky asset is traded in the capital market. He finds that when two agents who are motivated by different beliefs trade a risky asset, the question of which one is the buyer and which one is the seller depends critically on the degree of intertemporal substitution. To be specific, the more optimistic (pessimistic) agent will be the buyer (seller) when $0 < \rho < 1$, and the opposite is true when $\rho > 1$. From equation (30), one can easily see that there is no such decisive role of intertemporal substitution with the risk-free asset introduced here. The degree of intertemporal substitution does not matter as far as the demand for the risky asset is concerned. Nevertheless, it still plays an important role in agents' decisions on consumption-savings since intertemporal substitution affects the amount of riskless lending and borrowing (This will be shown later).

Let us turn to analyze how the trading volume of the risky asset is related to heterogeneous expectations among agents. Consider first the case where the trading volume is measured by the quantity $\sum_{i=1}^{n}(q^i)^2/2$. Although the trading volume, to be exact, should be defined by $\sum_{i=1}^{n}|q^i|/2$, the reason for using this alternative definition is to see its connection more clearly with the degree of heterogeneity among agents in general.[13] Substituting equation (30) into this definition will yield the following expression for the trading volume (T.V. for short).

$$T.V. \equiv \sum_{i=1}^{n}\frac{1}{2}\cdot\left(q^i\right)^2 = \frac{\sum_{i=1}^{n}\left(\mu^i-\bar{\mu}\right)^2}{2\cdot\left(\theta\cdot\upsilon\right)^2} - \frac{\sum_{i=1}^{n}\left(\mu^i-\bar{\mu}\right)\left(z^i-\dfrac{1}{n}\right)}{\theta\cdot\upsilon} + \frac{1}{2}\cdot\sum_{i=1}^{n}(z^i-\frac{1}{n})^2$$

$$= \frac{n\cdot Var^c(\mu^i)}{2\cdot(\theta\cdot\upsilon)^2} - \frac{n\cdot Cov^c(\mu^i,z^i)}{\theta\cdot\upsilon} + \frac{n}{2}\cdot Var^c(z^i) \qquad (31)$$

Equation (31) shows that the trading volume is increasing proportionately with the cross-sectional variance of subjective expectations, and with the cross-sectional variance of endowments. These results will verify the earlier conjecture about the connection between the trading volume and the degree of differences among agents. Moreover, in view of the result from equation (27), high trading volume turns out to be associated with low asset price.

Sheinkman and Xiong (2003) also derive a positive relationship between heterogeneity of beliefs and trading volume, but their model is characterized by the coexistence of high prices and high trading volume. They explain this phenomenon as follows: "Relatively more optimistic agents pay prices that exceed their own valuation of future dividends because they believe that in the future they will find a buyer willing to pay even more. This causes significant bubbles in asset prices even when small differences in beliefs are sufficient to

13) In fact, each component comprising the volume under the former definition may be thought as an increasing transformation of the corresponding trade under the original definition. Thus the two definitions contain the same information about components of the trading volume.

generate a trade. In equilibrium, bubbles are accompanied by large trading."

Meanwhile, the trading volume is negatively related to the cross-sectional covariance between the degree of optimism and the amount of endowments. In other words, a negative (positive) cross-sectional covariance will contribute to increasing (reducing) the volume of trades. The reason is that the absolute value of the demand function in equation (30) is larger when the two determinants of the trading volume change in opposite fashions.

Equation (30) also produces the results that trading volume of the risky asset is decreasing with risk aversion and the uncertainty of future dividends, but may be increasing or decreasing with the number of agents depending on whether the cross-sectional covariance term is negative or positive.

Let us turn to analyze the problem under the original definition of trading volume, T.V. $\equiv \sum_{i=1}^{n} |q^i|/2$. To highlight the effect of dispersion of different beliefs, I assume here that all agents are equally endowed ($z^i = 1/n$). In this case, the trading demand of agent i for the risky asset is $q^i = (\mu^i - \bar{\mu})/(\theta \cdot \upsilon)$.

Suppose that a continuum of investors are uniformly distributed with the expectation about y_1 ranging from $(\mu - \delta)$ to $(\mu + \delta)$. Then the probability density function is $f(\mu^i) = 1/2\delta$. And the cross-sectional mean and variance of μ^i can be calculated as follows.

$$\bar{\mu} = \int_{\mu-\delta}^{\mu+\delta} \mu^i \cdot \frac{1}{2\delta} \cdot d\mu^i = \mu \tag{32}$$

$$Var^c(\mu^i) = \int_{\mu-\delta}^{\mu+\delta} (\mu^i - \mu)^2 \cdot \frac{1}{2\delta} \cdot d\mu^i = \frac{1}{3} \cdot \delta^2 \tag{33}$$

In the domain of a continuum of investors, the trading volume is defined by $(1/2)\int |q^i|$, where $|q^i| = (\mu^i - \bar{\mu})/(\theta \cdot \upsilon)$ if $\mu^i > \mu$ and $|q^i| = (\bar{\mu} - \mu^i)/(\theta \cdot \upsilon)$ if $\mu^i < \mu$. Hence, the risky asset's volume of trades can be computed as follows:

$$\begin{aligned} T.V. &\equiv \frac{1}{2} \int_{\mu-\delta}^{\mu+\delta} |q^i| d\mu^i \\ &= \frac{1}{2} \left[\int_{\mu}^{\mu+\delta} \frac{\mu^i - \bar{\mu}}{\theta \cdot \upsilon} d\mu^i + \int_{\mu-\delta}^{\mu} \frac{\bar{\mu} - \mu^i}{\theta \cdot \upsilon} d\mu^i \right] = \frac{\delta^2}{2 \cdot \theta \cdot \upsilon} = \frac{3 \cdot Var^c(\mu^i)}{2 \cdot \theta \cdot \upsilon} \end{aligned} \tag{34}$$

Equation (34) will verify the earlier results that the trading volume is increasing proportionately with the cross-sectional variance of μ^i. Moreover, contrary to the argument of Dinh and Gajewski (2015) that the relationship between trading volume and heterogeneity of expectations is more concave than linear, it shows that the two are proportionally related. Moreover, the proportionality factor is decreasing with the degree of risk aversion, and the amount of risk as well. Thus, the more risk averse agents are, and the more uncertain asset payoffs are, the less agents trade the risky asset.

Trading Volume of the Riskless Asset (Riskless Lending or Borrowing)

To highlight the effect of increasing dispersion of beliefs on riskless lending or borrowing, let $z^i = (1/n)$ again. Then, rewriting equation (2) with respect to the amount of lending (or borrowing) and substituting equations (25), (30), and (24) gives

$$\begin{aligned} l^i &= \frac{1}{n} \cdot y_0 - p \cdot q^i - c_0^i \\ &= \left(\beta \cdot R_f\right)^{-1/\rho} \cdot \bar{\hat{c}}_1 - p \cdot \frac{\mu^i - \bar{\mu}}{\theta \cdot \upsilon} - \left(\beta \cdot R_f\right)^{-1/\rho} \cdot c_1^i \\ &= \left(\beta \cdot R_f\right)^{-1/\rho} \cdot \bar{\hat{c}}_1 - p \cdot \frac{\mu^i - \bar{\mu}}{\theta \cdot \upsilon} - \left(\beta \cdot R_f\right)^{-1/\rho} \cdot \left(c_1^{i*} + l^i \cdot R_f\right), \\ &\quad \text{where } \hat{c}_1^{i*} \equiv c_1^i - l^i \cdot R_f \end{aligned} \tag{35}$$

Rearranging equation (35) and using equations (23), (21), and (19) will yield the following relation:

$$\begin{aligned} (1 + \beta^{-1/\rho} \cdot R_f^{(\rho-1)/\rho}) \cdot l^i &= (\beta \cdot R_f)^{-1/\rho} \cdot \bar{\hat{c}}_1 - (\beta \cdot R_f)^{-1/\rho} \cdot c_1^{i*} - p \cdot \frac{\mu^i - \bar{\mu}}{\theta \cdot \upsilon} \\ &= (\beta \cdot R_f)^{-1/\rho} (\bar{\hat{c}}_1 - \hat{c}_1^{i*}) - p \cdot \frac{\mu^i - \bar{\mu}}{\theta \cdot \upsilon} \\ &= (\beta \cdot R_f)^{-1/\rho} \left(\frac{\frac{1}{n} \sum_{i=1}^{1} (\mu^i)^2 - (\mu^i)^2}{2 \cdot \theta \cdot \upsilon} \right) - \frac{\bar{\mu} - \frac{1}{n} \theta \upsilon}{R_f} \cdot \frac{\mu^i - \bar{\mu}}{\theta \cdot \upsilon} \end{aligned} \tag{36}$$

Hence, the riskless lending (or borrowing) of agent i can be derived as follows:

$$l^i = -A \cdot \left[(\mu^i)^2 - \frac{1}{n}\sum_{i=1}^{1}(\mu^i)^2 \right] - B \cdot \left(\mu^i - \bar{\mu} \right),$$

where $A \equiv \dfrac{1}{(2 \cdot \theta \cdot \upsilon)(\beta^{1/\rho} \cdot R_f^{1/\rho} + R_f)} > 0$ and

$$B \equiv \frac{\bar{\mu} - \frac{1}{n} \cdot \theta \cdot \upsilon}{R_f + \beta^{-1/\rho} \cdot R_f^{(2\rho-1)/\rho}} > 0 \tag{37}$$

In equation (37), if $\mu^i > \bar{\mu}$, then $(\mu^i)^2 > (1/n)\sum_{i=1}^{1}(\mu^i)^2$, and thus $l^i < 0$. Therefore, agent i, who is relatively optimistic, becomes a borrower. If $\mu^i < \bar{\mu}$, then $(\mu^i)^2 < (1/n)\sum_{i=1}^{1}(\mu^i)^2$, and thus $l^i > 0$. In this case, agent i, who is relatively pessimistic, becomes a lender.

Again, under the assumption that a continuum of investors are uniformly distributed with μ^i taking a value between $(\mu - \delta)$ and $(\mu + \delta)$, a calculation using the definition of the trading volume of the riskless asset yields

$$\begin{aligned}
T.V. &\equiv \frac{1}{2}\int_{\mu-\delta}^{\mu+\delta} \left| l^i \right| d\mu^i \\
&= \frac{1}{2} A \left[\int_{\mu-\delta}^{\mu} \left(\left(\frac{1}{2\delta}\int_{\mu-\delta}^{\mu+\delta} (\mu^i)^2 \cdot d\mu^i \right) - (\mu^i)^2 \right) d\mu^i \right. \\
&\quad \left. + \int_{\mu}^{\mu+\delta} \left((\mu^i)^2 - \left(\frac{1}{2\delta}\int_{\mu-\delta}^{\mu+\delta} (\mu^i)^2 \cdot d\mu^i \right) \right) d\mu^i \right]^{14)} \\
&\quad + \frac{1}{2} \cdot B \cdot \left[\int_{\mu-\delta}^{\mu} (\bar{\mu} - \mu^i) d\mu^i + \int_{\mu}^{\mu+\delta} (\mu^i - \bar{\mu}) d\mu^i \right] \\
&= \frac{1}{2} \cdot \left[A \cdot (2 \cdot \mu \cdot \delta^2) + B \cdot (\delta^2) \right] \\
&= \left[3A \cdot \mu + \frac{3}{2} B \right] \cdot Var^c(\mu^i)
\end{aligned} \tag{38}$$

Comparative static analyses with equation (38) will produce the following results: (i) As in the case of the risky assets, the trading

14) $\dfrac{1}{2\delta}\int_{\mu-\delta}^{\mu+\delta} (\mu^i)^2 \cdot d\mu^i = \mu^2 + \dfrac{1}{3}\delta^2.$

volume of the riskless asset is also increasing proportionally with the cross-sectional variance of μ^i. (ii) Since $\partial A/\partial v < 0$ and $\partial B/\partial v < 0$, the proportionality factor decreases with the uncertainty of future dividends. (iii) Since $\partial A/\partial\theta < 0$ and $\partial B/\partial\theta < 0$, the proportionality factor also decreases with the degree of risk aversion. (iv) Since $\partial A/\partial R_f < 0$ and $\partial B/\partial R_f < 0$ if $\rho \geq (1/2)$, an increase in the risk-free interest rate will reduce the trading volume, on the condition that $\rho \geq (1/2)$.

It is interesting to compare the last result with the well-known fact in the literature that savings increase (decrease) with the interest rate if ρ <(>) 1. The reason for the latter result is that the substitution effect is greater (less) than the income effect if ρ <(>) 1. This principle does not apply here in the range of $(1/2) \leq \rho < 1$. However, with the value of ρ above one, the two results are consistent with each other. By any means, unlike the case of the risky asset, the degree of intertemporal substitution plays an important role in determining the amount of riskless lending and borrowing. In turn, this will affect the optimal amount of consumption-savings.

V. SUMMARY AND CONCLUSION

This paper presents a theoretical model on asset pricing under heterogeneous beliefs. In particular, it extends a previous study of Cho (2001) into two different directions. First, following Abel (1990), it considers the case where both the riskless asset and a risky asset are traded in the capital market. In the model of Cho (2001), there is no riskless asset. Second, instead of the non-expected utility function of Epstein and Zin (1989) used in the preceding paper, this paper adopts the non-expected utility function that exhibits constant absolute risk aversion (CARA). The latter utility function makes it possible to obtain closed-form solutions of asset prices and trading volume while the Epstein and Zin utility function cannot with the existence of the riskless asset.

Major findings of this research are as follows. First, increased dispersion of beliefs about the mean of asset payoffs are associated with a higher risk-free rate and a lower risky asset's price. Abel (1990) obtained the same result using the time-additive expected utility function, but his closed-from solutions are independent of intertemporal substitution. Second, although the ex-post equity premium will rise as a consequence of the first result, the puzzle posed by Mehra & Prescott (1985) and Weil (1989) cannot be explained by heterogeneous expectations. In order for any model to resolve the puzzle at least partially, it should be able to generate a lower risk-free rate and a higher equity premium compared to the prediction of the Mehra and Prescott model. Third, the trading volume for both assets is increasing proportionately with the cross-sectional variance of beliefs. This result is contrary to the claim of Dinh and Gajewski (2015) that the relationship is non-linear. Fourth, the trading volume of the riskless asset decreases with the risk-free rate unless the intertemporal substitution parameter is less than 1/2.

In addition to these findings, the model of this paper obtains a lot more comparative statics results as by-products. (i) The risk-free rate increases with higher degree of intertemporal substitution, with lower degrees of risk and risk aversion, with lower value of the discount factor, with larger certainty equivalent of aggregate consumption growth, and with the population size. (ii) The risky asset's price is positively related with the current dividend, the utility discount factor, and with the number of agents having the intertemporal substitution parameter not greater than two. It also has a positive relationship with the mean of future dividend unless the intertemporal substitution is greater than 1. To the contrary, increasing risk and risk aversion will reduce the risky asset's price on the condition that the intertemporal substitution parameter does not exceed 2. (iii) The trading volume of the risky asset decreases with increasing risk and risk aversion. (iv) Larger trading volume of the riskless asset (riskless lending and borrowing) is associated with lower risk and risk aversion.

In conclusion, although the model of this paper obtains fruitful results in a static setting, it is worthwhile to be extended into a dynamic environment to be even more productive. Also, the model needs to be re-examined with the assumption that no short selling of the risky asset is allowed, since this assumption will alter the structure of asset demands. Finally, it is desirable to consider the case where multiple risky assets, instead of one risky asset, are traded. This will make it possible to derive heterogeneity of beliefs as a risk factor that explains security returns. All these subjects should be on the agenda of future research.

REFERENCES

Abel, A. B. (1990), Asset Prices under Heterogeneous Beliefs: Implications for the Equity Premium Puzzle, Working paper, Wharton School, University of Pennsylvania.

______ (1988), "Stock Prices under Time-Varying Dividend Risk: An Exact Solution in an Infinite-Horizon General Equilibrium Model," *Journal of Monetary Economics*, 22, 375-393.

Ajinkya, B. B., K. Atiase, and M. J. Gift (1991), "Volume of Trading and the Dispersion in Financial Analysts' Earnings Forecasts," *The Accounting Review*, 66, 389-401.

Anderson, E. W., E. Ghysels, and J. L. Juergens (2005), "Do Heterogeneous Beliefs Do Matter for Asset Pricing?" *Review of Financial Studies*, 18, 875-924.

Baker, S. D., B. Hollifield, E. Osambela (2016), "Disagreement, Speculation, and Aggregate Investments," *Journal of Financial Economics*, 19, 210-225.

Barsky, R. B. (1989), "Why don't the prices of stocks and bonds move together?," *American Economic Review*, 79, 1132-1145.

Basak, S. (1999), "A model of dynamic equilibrium asset pricing with heterogeneous beliefs and extraneous risk," *Journal of Economic Dynamics and Control*, 24, 63–95.

______ (2005), "Asset Pricing with Heterogeneous Beliefs," *Journal of Banking & Finance*, 29, 2849-2881.

Bhamra, H. S. and R.Uppal (2014), "Asset Prices with Heterogeneity in Preferences and Beliefs," *Review of Financial Studies*, 27, 519-580.

Biais, B. and P. Bossaerts (1998), "Asset Prices and Trading Volume in a Beauty Contest," *Review of Economic Studies*, 65, 307-340.

Chiarella, C. and X. He (2002), "Heterogeneous Beliefs, Risk and Learning in a Simple Asset Pricing Model," *Computation Economics*, 19, 95-132.

Chiarella, C., R. Dieci, and X. He (2007), "Heterogeneous Expectations and Speculative Behavior in a Dynamic Multi-Asset Framework," *Journal of Economic Behavior & Organization*, 62, 408-427.

Cho, J. (1992), "The Effects of Heterogeneous Beliefs on a Risky Asset's Price and Trading Volume," *Seoul Journal of Economics*, 5, 113-126.

______ (2001), "Intertemporal Substitution, Risk Aversion, and Asset Pricing under Heterogeneous Beliefs," *Korean Journal of Money & Finance*, 6, 1-20.

Dinh, T. H. and J-F. Gajewski (2015), "Trading Volume, Heterogeneous Expectations, and Earnings Announcements," *Journal of Behavioral Finance*, 16, 327-343.

Epstein, L. G. (1988), " Risk aversion and asset prices," *Journal of Monetary Economics*, 22, 177-192.

Epstein, L. G. and S. E. Zin (1989), “Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” *Econometrica*, 57, 937-969.

Hansen, S. L. (2015), “Cross-sectional Asset Pricing with Heterogeneous Preferences and Beliefs,” *Journal of Economic Dynamics and Control*, 58, 125-151.

Harris, M. and A. Raviv (1993), “Differences in Opinion make a Horse Race,” *Review of Financial Studies*, 6, 473-506.

Harrison, J. M. and D. M. Kreps (1978), “Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations,” *Quarterly Journal of Economics*, 92, 323–336.

Huang, S., Z. Qiu, Q. Shang, and K. Tang (2013), “Asset Pricing with Heterogeneous Beliefs and Relative Performance,” *Journal of Banking and Finance*, 37, 4107-4119.

Jarrow, R. (1980), “Heterogeneous Expectations, Restrictions on Short Sales, and Equilibrium Asset Prices,” *Journal of Finance*, 35, 1105-1113.

Jouini, E. and C. Napp (2006), “Heterogeneous Beliefs and Asset Pricing in Discrete Time: Analysis of Pessimism and Doubt,” *Journal of Economic Dynamics and Control*, 30, 1233-1260.

______ (2007), Consensus Consumer and Intertemporal Asset Pricing with Heterogeneous Beliefs,” *Review of Economic Studies*, 74, 1149-1174.

Karpoff, J. M. (1986), “A Theory of Trading Volume,” *Journal of Finance*, 41, 1069-1087.

______ (1987), “The Relation between Price Changes and Trading Volume: a survey,” *Journal of Financial and Quantitative Analysis*, 22, 109–126.

Kim, M. (1983), “Heterogeneous Expectations, Formation of Expectations and Predictive Accuracy: Evidence from Institutional Investor Survey,” Ph.D. dissertation, University of Pennsylvania.

Kim, M and J. Cho (2019), A Re-examination of the Equity Premium Puzzle and the Risk-Free Rate Puzzle in Korea, forthcoming in Asian Review of Financial Research.

Kimball, M. S. (1990), “Precautionary Saving in the Small and in the Large,” *Econometrica*, 58, 53-73.

Li, T. (2007), “Heterogeneous Beliefs, Asset Prices, and Volatility in a Pure Exchange Economy,” *Journal of Economic Dynamics & Control*, 31, 1697-1727.

Lintner, J. (1969), “The Aggregation of Investors’ Diverse Judgements and Preferences in Purely Competitive Security Markets,” *Journal of Financial and Quantitative Analysis*, 4, 347400.

Lucas, R. E. (1978),” Asset Prices in an Exchange Economy,” *Econometrica*, 46, 1429-1445.

Mehra, R. and E. Prescott (1985), “The equity premium puzzle,” *Journal of*

Monetary Economics, 15, 145-161.
Merton, R. (1980), "On estimating the expected return on the market: An exploratory investigation," *Journal of Financial Economics*, 8, 323–362.
Miller, E. (1977), "Risk, Uncertainty, and Divergence of Opinion," *Journal of Finance*, 32, 1151-1168.
Pfleiderer, P. (1984), The Volume of Trade and Variability of Prices: A Framework for Analysis in Noisy Rational Equilibria, Stanford University Working Paper.
Rubinstein, M. (1974), "An Aggregation Theorem for Securities Markets," *Journal of Financial Economics*, 1, 225-244.
Sheinkman, J. and W. Xiong (2003), "Overconfidence and Speculative Bubbles," *Journal of Political Economy*, 111, 1183-1217.
______ (2004), Heterogeneous Beliefs, Speculation and Trading in Financial Markets, Lecture Notes in Mathematics, Princeton University.
Svensson, L.E.O. (1989), "Portfolio choice with non-expected utility in continuous time," *Economics Letters*, 30, 313-317.
Varian, H. R. (1985), "Divergence of Opinion in Complete Markets: A Note," *Journal of Finance*, 40, 309317.
______ (1987), "Differences of Opinion in Financial Markets," Working Paper, University of Michigan.
Wang, J. (1993), "A Model of Intertemporal Asset Prices under Asymmetric Information," *Review of Economic Studies*, 60, 249–282.
Weil, P. (1989), "The Equity Premium Puzzle and the Risk-free Rate Puzzle," *Journal of Monetary Economics*, 24, 401-421.
______ (1993), "Precautionary Savings and the Permanent Income Hypothesis," *Review of Economic Studies*, 60, 367-383.
Williams, J. (1977), "Capital Asset Prices with Heterogeneous Beliefs," *Journal of Financial Economics*, 5, 219-239.
Zapatero, F. (1998), "Effects of Financial Innovations on Market When Beliefs are Heterogeneous," *Journal of Economic Dynamics and Control*, 22, 597-626.

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