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Markov Model of Word-of-Mouth Effect and Stock Market Participation*

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Abstract

The question of determinants of participation of stock market has long been a central question to financial economists. Most notably, Hong, Kubik, and Stein (2001) argue that social interactions affects the investment decision of potential stock market investors through two popular channels: word-of-mouth and pleasure-in-talk about stock market. In this paper, I extend Hong et al.'s model of social interactions to incorporate different effects of these two channels on stock market participation, conditioning on current market situation. The idea is intuitive: When potential investors observe current bull (bear) market, word-of-mouth and pleasure-in-talk effect would work positively (negatively) toward stock market participation due to increased number of peers who benefitted (lost their wealth) from bull (bear) market situation. In Markov chain process framework, I model stock market participation depending on current market situation and discuss empirical implications of my model.

Keywords: Markov chain, Word-of-mouth, pleasure-in-talk, stock market participation

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INTRODUCTION

Why people decide to join the stock market? This question has long been investigated by financial economists (Mankiw and Zeldes 1991; Heaton and Lucas 1999; Madrian and Shea 2000; Grinblatt and Keloharju 2001; Hong and Stein 2001; Brav et al. 2002; Duflo and Saez 2002; Vissing-Jorgensen 2002; Vissing-Jorgensen and Attanasio 2003) as well as practitioners. In a rational expectation framework, however, asset pricing models fail to explain the stock market participation of individuals. A famous equity premium puzzle shows that no conventional level of risk averseness could explain the revealed market risk premium (Mehra and Prescott 1985), leaving decision of stock market participation a puzzle. In pursue of answer to this question, researchers recently raise the issue of social interaction as a determinant of stock market participation (Glaeser et al. 1996; Bertrand et al. 2000). In this paper, I especially focus on the work of Hong et al. (2001: HKS thereafter), who show how social interactions affects the investment decision of potential stock market investors in their simple and precise model. They considered two channels through which social interaction can affect the decision of stock market participation: *word-of-mouth* (Banerjee 1992; Bikhchandani et al. 1992; Ellison and Fudenberg 1995) and *pleasure in talk about stock market* (Becker 1991). Due to these two effects, when more of his peers participate in the stock market, individual's *entry cost* of participating stock market is reduced, thus making *socials* more likely to join the stock market than *non-socials*.

In this paper, however, I argue that modeling of stock market participation through behavioral effects should reflect current market situation, which may affect the impact of these two channels on investment decision. It is well-understood that when potential investors observe current bull market, word-of-mouth and pleasure-in-talk effect would work positively toward stock market participation due to increased number of peers who benefitted from bull market situation. If current market is on its bear-situation, then potential investors who observe increased number of their peers who lost in the stock market may not be inclined to join the stock market that much. However, the distinction between the two channels is not made and the differential effect of them on stock market participation based on current market situation is not incorporated in the

HKS model. In addition, social interaction was assumed to have effect through entry cost *only*. That is, in the HKS model, potential stock market investors do not update his *belief* about future market situation after observing the fraction of stock market participants in their cohort. Probability of positive market net excess return in one period is *exogenously* given in HKS model. To overcome this issue and extend HKS to more realistic setting, I model decision of stock market participation incorporating the effect of current market situation on word-of-mouth and pleasure-in-talk. I also discuss empirical implications of my model in this paper.

To incorporate potential investors' belief into the model of investment decision, I model the stock market participation in a Markov process framework in this paper. In distinguishing the word-of-mouth and pleasure-in-talk effect, modeling by Markov process provides various benefits. Most importantly, the Markov process framework provides a simple and appropriate way of modeling the investment decision *conditioning on the current market situation*. Incorporating current market situation is important when only word-of-mouth effect is considered. In bull markets,[1] potential investors will observe more of his peers who already *earned* (*i.e., winners*) in the stock market. This will make socials more likely to join the stock market through word-of-mouth effect. But in a bear market situation, he is more likely to observe more of his peers who already *lost* their money in the stock market, which *negatively* affects his decision of stock market participation. Hence, in considering word-of-mouth effect, it is important to consider current market condition before I model the stock market participation. Markov model provides a good way of modeling such situation.

Another important point is that, in separating word-of-mouth effect from pleasure- in-talk, the fraction of *winners* in the cohort is more important than fraction of who merely *participate* in the stock market. If we want to consider pleasure-in-talk together, then fraction of participants can be important as in HKS. But by restricting the channel to only word-of-mouth, it is reasonable to assume that more of observed winners will positively affect participation decision: For example, if I observe that 90% of stock market participants

1) The terms of *bull* and *bear* markets are slightly different from their conventional meaning. I use the term 'bull (bear)' market to denote the stock market situation where a large number of investors who gained positive (negative) earnings in the stock market is observed.

are *losers* in my cohort, I will not be inclined to attend stock market even though 99% of my cohort are currently engaged in the stock market. Note that pleasure-of-talking effect can make me inclined to join the stock market in this situation, though. When everybody in the cohort is talking about stock market, I will be more inclined to join them to get the *pleasure-in-talk* about stock market. By considering only word-of-mouth, I exclude this possibility in this paper.

In the analysis, I do not separate socials and non-socials as HKS did. It is unreasonable to assume that non-socials will *never* have any kind of social interaction in their life.[2] It is more reasonable to think that non-socials do have social interaction but only up to a far less degree than socials do. Of course, it may be controversial that how much is "far less." But note that that's why I do not separate socials and non-socials here. While releasing unreasonable assumption of separating those two groups of possible investors, I still can show that social interaction does affect investment decision by using the Markov model explained in the next section. I will show that how the degree of such effect has influences on potential investors' investment decision.

Benefits of building Markov model and the differences of it from HKS are summarized in the below:

- I focus only on word-of-mouth effect, thus making it possible to distinguish the effects of two channels considered in HKS.
- I assume that potential stock market investors update their subjective expected probability of up- and down-market in one period through social interaction. Markov process provides a method of modeling such situation. That is, we now can investigate the *endogenous* probability of future market net excess return of potential investors.
- In this updating scheme, I successfully incorporate *prospect theory* (Kahneman and Tversky 1979) in the model. By doing so, I successfully model the differences in impact of positive and negative stock markets on investment decision through loss aversion.

2) HKS assumed this. The key in their explanation of peer effect is from different levels of entry cost for socials and non-socials. While the entry cost of socials decreases by observing more people who is joining the stock market in the cohort, that of non-socials are not affected by social interaction as implied by the name of *non-socials*.

- Instead of considering the fraction of who *participate* in the stock market in the cohort, I will consider the fraction of *winners* in the stock market in the potential investor's cohort.
- I do not separate socials and non-socials as in HKS. Instead, I will show the degree of effect that the social interaction has on the investment decision.
- I show that there exists a threshold level of social interaction that induces potential investors to join the stock market. Consistent with the practitioners' concern, I model that this threshold varies according to current market situation.
- By loosening the assumption of positive expected future return in HKS, I allow the different direction of investment, long vs. short, of potential investors. Also, I show the *size* of investment induced by social interaction is larger for long position than short position under some restriction.
- As in HKS, I can still use the framework of reduced *entry cost* in explaining the effect of social interaction on the decision of stock market participation. I start by reviewing HKS in the next section.

BRIEF REVIEW OF HKS MODEL

HKS is a one-period single asset model in which two possible future market excess returns are considered: The probability of decreased market return in one period, r_d (< 0), is π, thus increased market excess return in one period, r_u (> 0), has probability of $1 - \pi$. The model also assumes positive expected return in one period:

$$\pi r_d + (1-\pi)r_u > 0 \tag{1}$$

By solving maximization problem of power utility holder, equilibrium level of fraction invested in the stock market out of his wealth is given by

$$h^* = \frac{1-m}{mr_u - r_d} \tag{2}$$

where,

$$m = \left[\frac{-(1-\pi)r_u}{\pi r_d}\right]^{-\frac{1}{\gamma}}$$

Here, $m < 1$ holds by (1),[3] thus making equilibrium level of investment in (2) always *positive*.

Entry cost of stock market participation is assumed to be θc_i and $\theta c_i - B(p_s)$ for non-socials and socials, respectively. Here, c_i is the *idiosyncratic* participation-cost parameter, which is assumed to be distributed according to a cumulative distribution function $G(c_i)$ and θ is a *common* participation-cost parameter. $B(\cdot)$ is an increasing function of fraction of stock market participants in the cohort, p_s, so observing more of his peers who joined stock market will *reduce* the entry cost for socials. Through *individual rationality* conditions of socials and non-socials, HKS shows that the fractions of socials who joined stock market, p_s, is bigger than that of nonsocial-cohort, p_n.[4]

MODELING BY MARKOV PROCESS

I consider a single-period model with single risky asset in the economy. I assume zero risk-free rate for simplicity. Excess return on the market is assumed[5] to be r ($r > 0$) with unconditional probability of π and $-r$ with probability of $1 - \pi$. Investors have initial wealth of W_0 and their terminal wealth is W. I will use power utility function of $U(W) = W^{1-\gamma} / (1 - \gamma)$, where $\gamma > 0$. h denotes the fraction of wealth allocated to the stock market.

Now, I build some different setup with HKS. I assume that potential investors in sociable groups update their subjective expected probability of up- and down-market in one period after observing the fraction of *winners*. Let φ^W ($0 \le \varphi^W \le 1$) be the fraction of such winners out of the total number of stock-market participants in a given investor's cohort. Since φ^W uses total number of stock market

3) This assumption plays a crucial role by making equilibrium portion invested in the stock market always positive. So, the room for considering *short* position is removed by this assumption in the HKS model.

4) Details about comparing social and nonsocial fractions will be shown in Section 6.

5) The assumption of the same absolute value of excess returns in up- and down-market with different signs is actually a simpler one than in HKS. This reduces some tedious calculation while the loss from this assumption is trivial.

participants as denominator, $1 - \varphi^W$ denote the fraction of *losers* in her cohort.

Rather than assuming positive expected net excess return as in (1) of HKS, I will investigate the conditions which will affect the *direction* and participation in stock market. It is clear that people will not participate in the stock market by taking *long* position if the unconditional expected net excess return is negative. So, we have a *necessary* condition for non-negative investment in his *long* position, $\pi r - (1 - \pi) r \geq 0$, which gives us:

$$\pi \geq \frac{1}{2} \tag{3}$$

Let potential investor's conditional probabilities of future net excess returns given current market condition be:

$$\Pr(U_t | U_{t-1}) \equiv \pi^U = \frac{1}{2} + \alpha^U \varphi^W \tag{4}$$

$$\Pr(D_t | U_{t-1}) = 1 - \pi^U = \frac{1}{2} - \alpha^U \varphi^W \tag{5}$$

$$\Pr(D_t | D_{t-1}) \equiv \pi^D = \frac{1}{2} + \alpha^D (1 - \varphi^W) \tag{6}$$

$$\Pr(U_t | D_{t-1}) = 1 - \pi^D = \frac{1}{2} - \alpha^D (1 - \varphi^W) \tag{7}$$

U_t and D_t are the indicators for market condition at time t which denote the up-market and down-market, respectively while positive constants α^U and α^D respectively denote the coefficients for weights of φ^W and $1 - \varphi^W$. Note that π^U, $\pi^D \geq 1/2$ as implied by (4) and (6). Sufficient condition[6)] for (4)−(7) to be valid probabilities is:

6) Since (4)−(7) are probabilities which require non-negativeness and maximum value of one, the necessary conditions for the validity of my setting in (4)−(7) are: $0 \leq \alpha^U \leq 1/2\varphi^W$ and $0 \leq \alpha^D \leq 1/\{2(1 - \varphi^W)\}$. But this generates the problem that our α's depend on φ^W. Since my analysis is based on the *given* constant α, (8) serves as a good restriction.

$$0 \leq \alpha^U, \alpha^D \leq \frac{1}{2} \tag{8}$$

This setup of Markov process makes it possible that the potential investor *update* her subjective probability of future net excess return after observing the size of the fraction of winners in the group where she is joining in. Coefficients $a^{(\cdot)}$ works as a weight in updating her conditional probability with the observed fraction. Note that such weights depend on each given state in this model as is obvious by using two different a^U and a^D. This allows different updating schemes depending on current market situation: If she is more optimistic about the future when the given state is bull than she is pessimistic upon bear state, she will use the weighting skill of $a^U > a^D$. *Prospect theory* says that people have asymmetric attitude about risk: more pessimistic in bear-market situation than optimistic in a bull-market situation. To incorporate this into my model, I assume:

$$\alpha^U < \alpha^D \tag{9}$$

Intuitively, we think that when potential investors observe more winners in her groups, she will positively update her belief by increasing her conditional probabilities for future up-markets. The opposite would be applied when she observed more losers. My Markov setup clearly reflects this intuition as we see in the following:

$$\frac{\partial \pi^U}{\partial \varphi^W} = \alpha^U \geq 0 \tag{10}$$

$$\frac{\partial (1 - \pi^U)}{\partial \varphi^W} = -\alpha^U \leq 0 \tag{11}$$

$$\frac{\partial \pi^D}{\partial \varphi^W} = -\alpha^D \leq 0 \tag{12}$$

$$\frac{\partial (1 - \pi^D)}{\partial \varphi^W} = \alpha^D \geq 0 \tag{13}$$

Optimal fraction of investment of her wealth would be h^* which solves the expected utility maximizing problem of:

$$MaxE[U(W)] = \pi U(W^U) + (1-\pi)U(W^D) \tag{14}$$

where, $W^U \equiv W_0[(1-h) + h(1+r)]$ and $W^D \equiv W_0[(1-h) + h(1-r)]$.

First order necessary condition is also sufficient by the risk-averseness of the given utility function. Solution, as denoted by *unconditional* probability of π is as follows:

$$h^* = \frac{1-m}{r(1+m)} \tag{15}$$

where,

$$m = \left[\frac{1-\pi}{\pi}\right]^{\frac{1}{\gamma}} \tag{16}$$

Note that by (3), $m < 1$ holds when the potential investor join the stock market by taking *long* position.

In the next subsection, I will check if this solution has the reasonable features that fit some of our intuitions discussed earlier.

HOW THE FRACTION OF WINNERS OBSERVED IN THE COHORT AFFECT OPTIMAL INVESTMENT DECISION?

In this section, I will investigate the behavior of stock-market participant when he does *not* condition on current market situation while his expectation of future net excess return of market is given by the conditional probabilities of (4)−(7). Similar study for conditioning case will be made in the subsequent sections.

By straight calculation with the assumption of steady-state distribution of unconditional probability π, we can get the expression of unconditional probability of future up-market by conditional probabilities of (4)−(7).

$$\pi = \frac{1-\pi^D}{2-\pi^U-\pi^D} \tag{17}$$

By (17), m in (16) can also be expressed by *conditional* probabilities:

$$m = \left[\frac{1-\pi^U}{1-\pi^D}\right]^{\frac{1}{\gamma}} \tag{18}$$

Intuition says that the bigger the fraction of winners in the stock market in the cohort is observed, the more likely the potential investors actually join the stock market. This intuition can easily be checked by investigating the sign of $\partial h^* / \partial \varphi^W$.

Proposition 1 *If the potential investor observes more of stock-market winners in the cohort that he is joining, he will be more likely to increase his portion of wealth invested in the stock-market. That is,*

$$\frac{\partial h^*}{\partial \varphi^W} > 0 \tag{19}$$

holds.

Proof 1 *By (15) and (16),*

$$\frac{\partial h^*}{\partial m} = \frac{-2}{r(1+m)^2} < 0 \tag{120}$$

From (18), (5) and (7),

$$\frac{\partial m}{\partial \varphi^W} = \frac{1}{\gamma}\left[\frac{1-\pi^U}{1-\pi^D}\right]^{\frac{1}{\gamma}-1}\left[\frac{-\alpha^U\left(\frac{1}{2}-\alpha^D(1-\varphi^W)\right)-\alpha^D\left(\frac{1}{2}-\alpha^U\varphi^W\right)}{\left(\frac{1}{2}-\alpha^D(1-\varphi^W)\right)^2}\right]$$

$$= \frac{1}{\gamma}\left[\frac{1-\pi^U}{1-\pi^D}\right]^{\frac{1}{\gamma}-1}\left[\frac{-\frac{1}{2}(\alpha^U+\alpha^D)+\alpha^U\alpha^D}{\left(\frac{1}{2}-\alpha^D(1-\varphi^W)\right)^2}\right]$$

So,

$$sign\left(\frac{\partial m}{\partial \varphi^W}\right) = sign\left(-\frac{1}{2}(\alpha^U+\alpha^D)+\alpha^U\alpha^D\right)$$

$$= sign\left[-\frac{1}{2}\{\alpha^U(1-2\alpha^D)+\alpha^D\}\right] \tag{21}$$

which is always negative by (8). By combining (21) with (20), we get (19), which completes the proof.

Now, I investigate the *threshold level* of the fraction of winners in the cohort that will induce potential investor to invest in the stock market when he observes bigger portion of winners in his cohort that exceeds such threshold. That is, I will check if there exists φ^{W*} such that potential investors will join the stock market if he observes $\varphi^{W} > \varphi^{W*}$.

Proposition 2 *There exists φ^{W*} such that $h^* > 0$ if and only if $\varphi^W > \varphi^{W*}$. And that threshold level is*

$$\varphi^{W*} = \frac{\alpha^D}{\alpha^U + \alpha^D} \tag{22}$$

Proof 2 *From (15), it is clear that $h^* > 0$ if and only if $m < 1$ and by (18), $m < 1$ if and only if $1 - \pi^U < 1 - \pi^D$. So, by (5) and (7), $m < 1$ is equivalent to $1 - \pi^U - (1 - \pi^D) = a^D - \varphi^W(a^D + a^U) < 0$. By letting $\varphi^{W*} \equiv a^D/(a^U + a^D)$, I complete the proof.*

This proposition provides an interesting and intuitively appealing fact. When we say that social interaction affects the decision of potential investors whether he will join the stock market or not, it is reasonable to think that he should observe *sufficiently* large ratio of people in his cohort who already got successful performances in the stock market. Proposition 2 gives us the lower bound that such *sufficiently* big ratio should exceed. Note that this threshold level consists of a^U and a^D, which are the marginal effects of fraction of successful investors in the cohort on updating the beliefs of potential investor.

If the potential investor is more pessimistic, thus has bigger a^D, then he should observe bigger ratio of successful market participants in his cohort before he decides attending stock market. Proposition 2 shows such intuition is correct by producing *higher* threshold level for more *pessimistic* potential investors.

It looks counter-intuitive that the potential investor would take *short* position (*i.e.*, $h^* < 0$) if he observes *smaller* fraction of winners than threshold given in proposition 2. To see this, note that the proof of proposition 2 gives us

$$\pi^U > \pi^D \;\Leftrightarrow\; \varphi^W > \varphi^{W*} \tag{23}$$

Equivalent relation of (23) says that *the threshold level of (22) in proposition 2 is actually the threshold level of investor* ***sentiment*** *to make a potential investor be more optimistic in a bull market than he is pessimistic in a bear market* ($\pi^U > \pi^D$), which leads to take *long* position. If he observes *smaller* fraction than the threshold of (22), *pessimism* in a bear market will dominate his optimism in a bull market, thus making pessimism more dominant. It is *not* counter-intuitive that an investor who has a *pessimistic* view of future market would take *short* position.

Note that the result in proposition 2 is from the interpretation of *conditional* probabilities in terms of *unconditional* probability. In later sections, we will see *threshold level is* ***lower*** *given* ***bull*** *current market situation than the threshold level of (22), thus making people* ***more likely to join the stock market when the current market is in its bull status***.

WHEN THE POTENTIAL INVESTOR'S DECISION OF JOINING STOCK MARKET DEPENDS ON CURRENT MARKET SITUATION

Now, I consider the situation where the potential investors consider current market situation to decide whether or not he will join the stock market. Effect of social interaction in this case also varies depending on current market situation since current market situation affects the threshold level of successful stock market participants in the cohort as in proposition 2.

When the current market is in a bull-market status

It is reasonable to assume that more people will be induced to join the stock market when current market is in bull-market status since he may observe more people in the cohort who already been successful in the stock market. In this case, his updating of belief is made through the conditional probabilities given in (4) and (5). Solving his problem of maximizing expected utility as (14) in the previous section gives us

$$h^{U*} = \frac{1 - m^U}{r(1 + m^U)} \tag{24}$$

where,

$$m^U = \left[\frac{1 - \pi^U}{\pi^U}\right]^{\frac{1}{\gamma}} \tag{25}$$

As in the previous section, I can easily derive similar result in this case.

Proposition 3 *When the current market is in bull status, if the potential investor observes more of stock-market winners in the cohort that he is joining, he will be more likely to increase his portion of wealth invested in the stock-market. That is,*

$$\frac{\partial h^{U*}}{\partial \varphi^W} > 0 \tag{26}$$

holds.

Proof 3 *By (24) and (25),*

$$\frac{\partial h^{U*}}{\partial m^U} = \frac{-2}{r(1 + m^U)^2} < 0 \tag{27}$$

From (25), (4) and (5),

$$\frac{\partial m^U}{\partial \varphi^W} = \frac{1}{\gamma}\left[\frac{1 - \pi^U}{\pi^U}\right]^{\frac{1}{\gamma}-1}\left[\frac{-\alpha^U}{\left\{\frac{1}{2} + \alpha^U\varphi^W\right\}^2}\right] < 0 \tag{28}$$

By combining (27) and (28), we get (26), which completes the proof.

Now, I investigate the *threshold level* of the fraction of winners in the cohort. Intuition says that when conditioning on current *bull* market situation, the potential investor will be more likely to attend the stock market since he can observe that more people in his cohort earned positive gain in the stock market. Thus, I expect the threshold level will be reduced when the current market is in bull status.

Proposition 4 *When conditioning on current bull market situation, threshold level of wealth which induces potential investor to be involved in stock market will be less than that of (22) for unconditioning case. Threshold level* φ_U^{W*} *such that* $h^{U*} > 0$ *if and only if* $\varphi^W > \varphi_U^{W*}$ *is zero in this case.*

Proof 4 *From (24), it is clear that* $h^{U*} > 0$ *holds if and only if* $m^U < 1$ *and by (25),* $m^U < 1$ *if and only if* $1 - \pi^U < \pi^U$*. This is equivalent to* $\pi^U > 1/2$*, which gives us* $\varphi_U^{W*} = 0$

Reduced threshold level in this case is intuitively appealing.[7)] When the current market is in a bull status, potential investors can observe more people in the cohort who already earned positive gains in the stock market. In addition, relatively wide spread optimism about the stock market makes him more comfortable about investment in stock market.

When the current market is in a bear-market status

When the current market is in bear status, it is more likely that the potential investors observe more people who lost their money by investing in the stock market. This has a negative impact in his decision of stock market participation. The following propositions show that potential investors will join by *shorting* in the stock market rather than taking long position when the current market is in bear status. Relevant probabilities in this case will be (6) and (7). I get the following equilibrium level of investment by solving the problem of maximizing expected utility.

$$h^{D*} = \frac{m^D - 1}{r(1 + m^D)} \tag{29}$$

where,

7) The zero threshold value may not seem to bear much of economic intuition but is obtained because potential investors' expectation of future returns is modeled as a linear function of fraction of winners in equations (4) to (7). The key implication of the proposition is the fact that the threshold is reduced when the current bull market status is considered compared to the case when it is not.

$$m^D = \left[\frac{1-\pi^D}{\pi^D}\right]^{\frac{1}{\gamma}} \tag{30}$$

Note that

$$h^{D^*} \leq 0$$

since $m^D \leq 1$ which is the result of $\pi^D \geq 1/2$ from (6). This implies that when the current market is in bear status, the potential investors will take *short* position or will not involve in the stock market. The following proposition shows that by observing more fractions who *failed* in the stock market, the amount of short position is *reduced* while observing bigger fraction for *successful* participants in the cohort *increases* his short position.

Proposition 5 *When the current market is in bear status, if the potential investor observes more of stock-market* ***winners*** *in the cohort that he is joining, he will be more likely to* ***increase*** *his portion of wealth* ***shorted*** *in the stock-market. If he observes more of stock-market* ***losers****, then he will* ***reduce*** *the amount of* ***short*** *position. That is,*

$$\frac{\partial h^{D*}}{\partial \varphi^W} < 0$$

$$\frac{\partial h^{D*}}{\partial (1-\varphi^W)} > 0 \tag{31}$$

holds.

Proof 5 *By (29) and (30),*

$$\frac{\partial h^{D*}}{\partial m^D} = \frac{-2m^D r}{r(1+m^D)^2} < 0 \tag{32}$$

From (30), (6) and (7),

$$\frac{\partial m^D}{\partial \varphi^W} = \frac{1}{\gamma}\left[\frac{1-\pi^D}{\pi^D}\right]^{\frac{1}{\gamma}-1}\left[\frac{\alpha^D}{\left\{\frac{1}{2}+\alpha^D(1-\varphi^W)\right\}^2}\right] > 0 \tag{33}$$

By combining (32) and (33), we get (31), which completes the proof.

Effect of social interaction on the size of the position

In the previous section, I investigated the effect of social interaction on *direction* of the position that potential investors will take. Now, we observe the *size* of the position when potential investor's decision of joining stock market depends on current market situation.

Taking short position is thought as more risky than taking long position. It is from the limit of possible loss of investment: The potential maximum loss by taking long position is limited by the amount that he paid to buy the stocks, but in short position, theoretically, the possible loss is infinite. Thus, intuition says that the absolute amount of long position that the potential investor takes when the current market is in a bull status will be *larger* than the amount of short position that he would take when the current market is in bear status. Proposition 6 shows this.

Proposition 6 *There exists a threshold level φ^{W**} such that the* ***size*** *of investment of* ***long*** *position when the current market situation is* ***bull*** *is* ***larger*** *(smaller) than the size of short position taken when the current market is* ***bear*** *if $\varphi^{W} > (<) \varphi^{W**}$. And that threshold level is*

$$\varphi^{W**} = \frac{\alpha^D}{\alpha^U + \alpha^D} \tag{34}$$

Proof 6 *By (24), (25), (29) and (30),*

$$h^{U*} + h^{D*} = \frac{1}{r}\left[\frac{2(m^D - m^U)}{(1+m^U)(1+m^D)}\right]$$

Thus, with (25), (30), (4) and (6),

$$\begin{aligned} sign(h^{U*} + h^{D*}) &= sign(m^D - m^U) = sign(\pi^U - \pi^D) \\ &= sign(\varphi^W(\alpha^U + \alpha^D) - \alpha^D) \end{aligned}$$

which completes the proof.

Proposition 6 says that when the potential investors observe bigger portion who earned in the stock in his cohort, his amount of taking long position is bigger than his amount of short position when the current market situation is not favorable. But when he observes *less* fraction of winners in his cohort when the current market is in a bear status, he takes short position more *aggressively*.[8] Prospect theory supports this result: Observing smaller fraction of winners in his cohort in a bear current market situation makes people more *pessimistic* than observing larger fraction of winners in a current bull market situation makes people more optimistic, thus leads to more aggressive short position when smaller fraction of winner is observed in an unfavorable market situation.

ARE SOCIALS MORE LIKELY TO JOIN THE STOCK MARKET THAN NON-SOCIALS?

In the previous sections, we saw the effect of social interaction on stock market participation by observing a fraction of his wealth invested in the stock market, h. In this section, we will see this peer effect through *individual rationality*. Difference with HKS is that *I will use* ***updating scheme of beliefs*** *rather than entry cost as in HKS*.

Recall (15). By plugging (15) back into $E\,[U\,(W)]$ in (14), we get the value function for the stock market participants:

$$V_P = \frac{KW_0^{1-\gamma}}{1-\gamma} \tag{35}$$

where,

$$K = \left[\frac{2}{m+1}\right]^{1-\gamma} [(1-\pi)m^{1-\gamma} + \pi] \tag{36}$$

And m is given in (16).

From

8) Since the model assumes nothing about the performance of investment, the reason of observing less fractions of winners in a down market is due to small amount of short-selling.

$$V_P = U(W_P) = \frac{W_P^{1-\gamma}}{1-\gamma}$$

we get the certainty equivalent of wealth, W_p, for a stock market participants,

$$W_P = K^{\frac{1}{1-\gamma}} W_0 \tag{37}$$

As in HKS, let θ and c_i be *common* and *idiosyncratic* participation-cost parameters, respectively. c_i is assumed to be distributed according to a cumulative distribution function $G(c_i)$. It is important to note that I use θc_i as the cost for entering stock market for *both socials and non-socials.* This is different from HKS since they use different entry cost for different cohorts.

Individual rationality condition says that the potential investors participate stock market only if

$$W_P - W_0 = \left(K^{\frac{1}{1-\gamma}} - 1\right) W_0 > \theta c_i \tag{38}$$

So, the fraction of stock market participants in the cohort is given by

$$G\left(\frac{\left(K^{\frac{1}{1-\gamma}} - 1\right) W_0}{\theta}\right) \tag{39}$$

Proposition 7 *If $\gamma > 1$, **the bigger** the fraction of winners in the cohort, **the more** potential investors will join the stock market. That is,*

$$\frac{\partial G(\cdot)}{\partial \varphi^W} > 0 \tag{40}$$

holds when $\gamma > 1$ where $G(\cdot)$ is from (39).

Proof 7 *Since $m^{-\gamma} = \pi/(1-\pi)$ from (16), K in (36) can be rewritten* *as $K = 2^{1-\gamma} \pi (m + 1)^{\gamma}$, which clearly shows that $\partial K / \partial m > 0$. In the proof of proposition 1, I showed $\partial m/\partial \varphi^W < 0$. Thus, $\partial K/\partial \varphi^W < 0$. If $\gamma > 1$, $\partial K^{1/(1-\gamma)}/\partial K = 1/(1 - \upsilon)K^{\gamma/(1-\gamma)} < 0$, which completes the proof.*

Proposition 7 clearly shows that observing more winners in the cohort induces more people to participate in the stock market. This also supports the conclusion of HKS: Socials are more likely to join the stock market than non-socials.

CONCLUSION

In this paper, I argue that modeling of stock market participation through behavioral effects should reflect current market situation, which may affect the impact of these two channels on investment decision. I model stock market participation in Markov chain process framework incorporating the effect of current market situation on word-of-mouth and pleasure-in-talk. I also discuss empirical implications of my model in this paper.

My model is based on the assumption of power utility function and, hence, is sharing the problems that power utility function has. However, my Markov model can be widely applied to other investment behavior rather than social interaction by allowing *various interpretation of φ^W*.[9] In my Markov model, I defined φ^W as a fraction of winners in the cohort. We may interpret it as an *index of market sentiment or as current market situation* itself.

Even non-socials may have *updating belief* scheme. By allowing other interpretation of φ^W, we can model this non-social's behavior, too: Instead of having some social interactions, non-socials will get much of the information about current market situation from TV, internet or any other sources. In this case, for example, interpretation of φ^W as a *current stock market index* helps model non-social's updating scheme.

Interesting interpretation of φ^W will be related to the *number of analysts on a given stock*.[10] This also gives us an opportunity of related *empirical study*. Updating of this paper will be made exactly

9) I thank Bing Han for pointing this out.

10) Some modification will be necessary for this since φ^W should be a *ratio* which has a value between 0 and 1.

on this point in the recent future.

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