

## Object-Oriented Simulation Model for Tandem Tollbooth Operations using Pro-Model<sup>\*</sup>

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### Abstract

This paper presents a new object-oriented simulation model to describe tandem tollbooth system in order to evaluate the operational performance of toll plaza facilities. This microscopic traffic simulation model is used to enhance planning, design, operations, and management of traffic operation at toll plazas. Computational result shows that lower reaction time ratio increases the throughput capacity of tandem tollbooth.

Keywords: Object-oriented Modeling, Simulation, Tandem Tollbooth, Traffic Flow

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## INTRODUCTION

Traffic volume has increased continuously due to increasing number of vehicles and introduction of the five working days a week policy. Congestion on a road and pollutants in the air due to heavy traffic bring about enormous social and economic costs. Toll plaza is the facility to collect road service fares. Toll plaza interrupts traffic flows and causes delays in collecting service fee, since all the vehicles should stop at the toll booth to pay their tolls and wait at the queue.

The number of tollbooth facilities is a critical issue to decrease waiting time in a queue and to operate toll plaza more efficiently. Increasing the number of toll booths widely sometimes may not be a feasible plan because it requires huge investment for additional land or it is constrained by buildings or other physical barriers. These barriers may make the cost of adding toll lanes prohibitive (Hall and Daganzo 1983). Tandem tollbooth, which means two toll collectors in a single toll lane both serving alternating sets of vehicles simultaneously (Hall and Daganzo 1983), is an easy and cost efficient plan compared with electronic toll collection system (ETCS) because ETCS requires much higher installment cost and may cause vehicles without ETC device to wait longer due to reduced number of traditional tollbooth. Furthermore, serialization of traffic flow by tandem tollbooth can prevent several accidents by sudden lane change or careless toll selection decision. Therefore, tandem tollbooth could increase the capacity of toll plaza and may decrease the probability of accidents by making traffic stream smoother.

In this paper, we present an object-oriented simulation model to evaluate the operation of tandem tollbooth according to the reaction and service time. The purpose of this research is to build a foundation to analyze tandem toll booth operation and performance. To our knowledge, this research is one of the first to apply object-oriented stochastic tandem queue simulation to the tollbooth system. Although there are several papers on tandem queue, most of them are in the field of manufacturing. Compared with Hong et al. (2009), the approach of this paper is more flexible and realistic in that this paper can deal with the operation of tandem tollbooth at non-peak time. This paper shows that in the tandem toll booth the ratio of reaction mean time to service mean time affects the system

throughput more than the variance of the service time itself.

The rest of this paper is organized as follows. In section 2, we present literature review. In section 3, we give a definition of tandem tollbooth. This definition is used in Section 4 to design object-oriented simulation model for tandem tollbooth. Section 5 reports computational experiments for simulation model. Finally, conclusion and suggestions for future research are discussed in the last section.

## LITERATURE REVIEW

There are several attempts to describe toll plaza using simulation. First of all, Ozbay, Bartin, and Mudigonda (2006) presents the development of a microscopic simulation model of the New Jersey Turnpike using Paramics microscopic simulation software to model the complex lane changing and lane choice behavior at toll plazas. "The model is validated and calibrated using the detailed vehicle-by-vehicle toll data at each toll plaza in the NJTPK. The dataset includes the entry toll plaza, exit toll plaza, entry and exit times, entry and exit lanes for each vehicle" (Ozbay, Bartin, and Mudigonda 2006). Moreover, Correa, Metzner, and Nino (2004) suggest a stochastic simulation model and experimentation for a toll system developed for a highly concurred highway toll in Venezuela. Correa, Metzner, and Nino (2004) state that "The main objective of TollSim is to start building an infrastructure and infrastructure to be used in a decision support system for management of traffic related issues in a technologically excluded region. The model was developed using an object-oriented approach." Al-Deek, Mohamed, and Radwan (2000) present a discrete-event stochastic object-oriented microscopic simulation model to enhance planning, design, operation, and management of transportation facilities. So far many papers have adopted simulation method to analyze the effect of Electronic Toll Collection at the toll plazas or the operation of tollbooth. For example, when a toll plaza is designed, the right number of tollbooths to minimize congestion is a critical issue on operation. To determine the optimal number of tollbooths, some papers created a model for traffic flow and toll collection in a toll plaza. Another case is for lane selection and lane-changing at toll plazas. Mudigonda, Bartin, and Ozbay (2009) suggest "the microscopic simulation of

the operations at a toll plaza to understand the driver behavior at the toll plaza.” Their study involves “the development of the new methodology for the microscopic simulation of toll plaza operations” (Mudigonda, Bartin, and Ozbay 2009).

There are only a few papers on tandem tollbooth. Hall and Daganzo (1983) is the first paper dealing with tandem tollbooths at the toll plaza. This paper defines a tandem toll booth as “two toll takers in a single toll lane both serving alternating sets of vehicles simultaneously.” In this paper, authors emphasize the benefit of tandem tollbooth in areas which have physical restraint to extend. They indicate that “many toll plazas are constrained in width by buildings or other physical barriers. These barriers may make the cost of adding toll lanes prohibitive. One method for increasing the capacity of a toll facility without increasing its width is to use tandem toll booths (Hall and Daganzo 1983). Other papers deal with general tandem queues in production systems and communication networks, not specified in toll plaza. First, Brandwajn and Jow (1988) analyse open systems of tandem queues with blocking caused by finite buffers between servers. However, this paper presumed that tandem queuing system permits its complete service when entity passes all of serial facilities sequentially. In contrast, a driver in a tollbooth system can complete toll payment by passing only one of several serial tollbooths in a tandem tollbooth system. Another author directly deals with tandem queuing phenomenon especially at the toll plaza is Hong et al. (2009). They suggest the mathematical model using Max-Plus Algebra theory. This paper developed an efficient recursive model to represent tandem tollbooth at peak time. However, the disadvantage of this model is that arrival rate should be always greater than service rate. In reality, Hong et al. (2009)’s mathematical model may not be practically applicable because of strong assumptions on arrival rate and driver behavior, particularly in the situation that arrival rate is not steady or high enough to make queue full or in the case that driver learning effect is too slow to use tandem tollbooth so that simultaneous usage decrease.

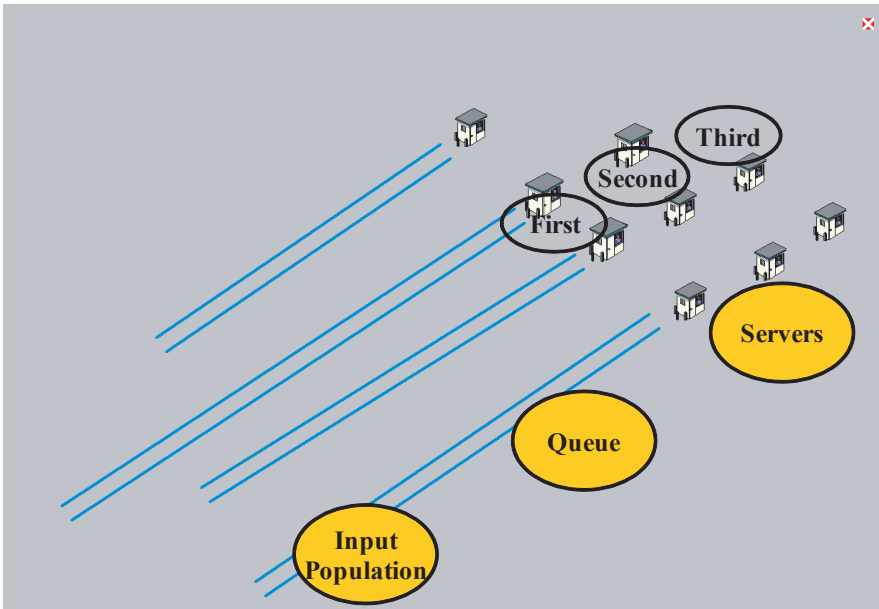
To reflect more realistic conditions and make tollbooth model flexible, a simulation model should consider stochastic and dynamic arrival of vehicles and physical structure of tollbooth such as length or width of vehicle and tollbooth. Object-oriented approach is one of proper methods to address the complexity of these constraints because object-oriented approach considers the problem not as a set

of functions but primarily as a set of related objects. Motivated by the mathematical model described in Hong et al. (2009), this paper focuses on the object-oriented simulation modeling method to describe the tandem tollbooth system. Furthermore, the experimental results of this paper confirms the result of mathematical model at the peak time and the results of Hong et al. (2009) paper are used to validate the tandem tollbooth in this research.

## MODEL STRUCTURE

### Model Concept and Assumption

To describe a tandem tollbooth system, we divide it into three components: input population, queue, and servers as shown in Figure 1. The first and second tollbooths also are defined in Figure 1. We make several assumptions for each component to reflect real-world environments and conditions. First,  $\lambda$  (mean arrival rate) is assumed to exceed  $\mu$  (mean service rate) of a single booth since a tandem tollbooth is generally open at peak time. Thus, vehicles are



**Figure 1. Layout of a Tandem Tollbooth System**

full in a queue when tandem tollbooth is open. This means that we don't need to consider arrival rate seriously because there are a lot of vehicles in a queue when we operate tandem tollbooth generally. Second, we assume queues with infinite buffers since a matter of interest is not a queue length but waiting time of the  $k^{th}$  vehicle in the system. Finally, tandem tollbooth facility follows FIFO (First In, First Out) rule as service discipline and consists of  $N$  number of service facility.

Tollbooth operation has its unique features. First, vehicles in a tollbooth system can complete toll payment by passing only one of several serial tollbooths in a tandem tollbooth system while a customer in general queuing systems is required to be served by all of serial facilities sequentially. Second, following drivers cannot pass over their preceding vehicle and may suffer delays due to the preceding driver who is not served completely because rear vehicles in a lane are blocked. Figure 1 illustrates the layout of a tandem tollbooth system.

## MODEL DEVELOPMENT

### Modeling Elements

The ProModel is one of the most widely used powerful easy-to-use simulation tools for modeling all types of management systems, ranging from microscopic queuing systems to large-scale supply chain systems. ProModel is a Windows based system with an intuitive graphical interface and object-oriented modeling method. The modeling elements of ProModel provide the building blocks for the physical and logical components of the system being modeled. The modeling elements of the system may be referenced either graphically or by name.

#### *Arrivals*

Deterministic or stochastic arrivals can be modeled using this element. Built-in or user defined distributions are used to define inter-arrival times and quantities. An arrival Element is defined by specifying the following information: number of entities per arrival, frequency of the arrivals, location of the arrival, time of the first arrival, total occurrences of the arrival.

### *Locations*

To represent fixed locations used in the system (e.g. facilities, queues, arrival, areas, exit areas), location elements can be used as either single unit locations (e.g. a single machine) or multi-unit locations (e.g. a group of similar machines performing the same operation in parallel). In addition, attributes can be defined for locations and contain real or integer values.

### *Entities*

An entity element refers to the items being processed in the system. Entities include raw materials, a piece of parts, loads, vehicles. Entities may be assigned attributes that can be tested in making decisions or for gathering specialized statistics. The graphic of an entity can be changed as a result of an operation to show the physical change during the animation.

### *Processing*

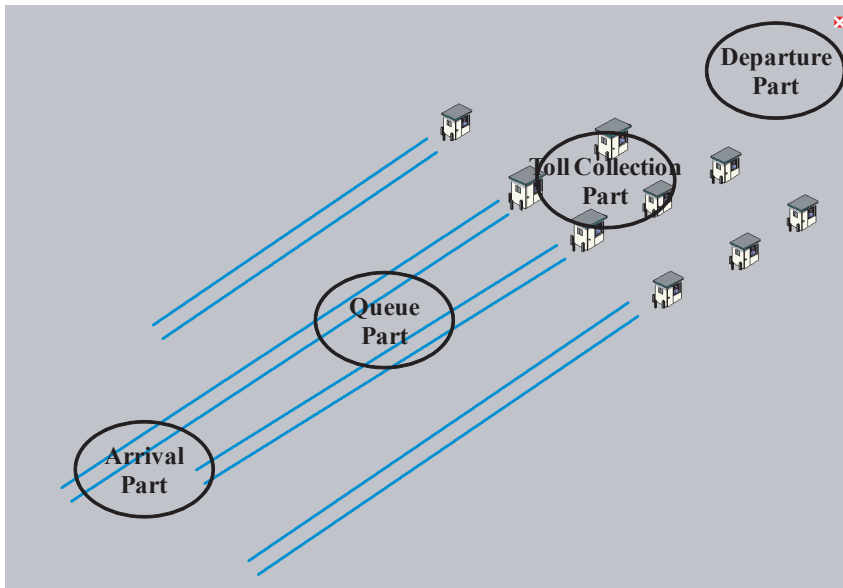
This element defines the processing sequence and flow logic of entities between locations. The operation logic, service times at locations, routing conditions, move times or requirements can be described using the processing element.

## **Modeling and Algorithms**

The simulation algorithm consists of four sequences: arrival part, queue part, toll collection part, and exit part. Besides, the components used in this paper are illustrated as follows.

### *Step 1: Arrival Part*

Tandem tollbooth is generally open at peak time. Thus, vehicles form a queue when tandem tollbooth is open. To represent this, we warm up simulation model before collecting statistical data. To assign reaction and service time to drivers, we generate random variables for reaction time and service time for car entities. For this reason, service time,  $S_k$ , and reaction time,  $R_k$ , for a vehicle are generated when car entities are generated, respectively. Mean and standard deviation of each distribution come from Al-Deek, Mohamed, and Radwan (2000), Hong et al. (2009) and Sadoun (2005). And then, these car entities are routed to each tollbooth queue. Arrivals are stochastic.



**Figure 2. ProModel's Output Data Representation**

### *Step 2: Queue Part*

To present “Stop and Go” phenomenon, we utilize lengthwise accumulating conveyers, which means that conveyor accumulates entities based on length rather than width of entities. Actually, in the tollbooth queue area, lane changes are not common due to severe congestion. Thus, it is reasonable to represent queue area by using accumulating conveyor.

Normally, a conveyor is a location that simulates the movement of entities on either an accumulating or non-accumulating conveyor. For accumulating conveyors, if the lead entity is unable to exit the conveyor, trailing entities queue up behind it. For non-accumulating conveyors, if the lead entity comes to a stop, the conveyor and all other entities stop (Benson 1997). In this paper, to describe traffic movement in the queue, we select an accumulating conveyor since the traffic of conveyed entities needs to be accumulated, in order to avoid collisions. In addition, we choose FIFO (First-in, First-Out) rules. Therefore, entities can only enter an accumulating conveyor at the beginning and leave at the end. Accumulation conveyor tracks are used for buffering vehicles entered into the queue. Movement on a conveyor is based on the length and speed of the conveyor, as



well as the length of the entity. The move time for an entity on a conveyor is calculated using the following formula:  $\text{Time} = (\text{Conveyor Length} - \text{Entity Length}) / \text{Conveyor Speed}$

### *Step 3: Toll Collection Part*

To represent several unique phenomena occurred in toll facilities, we build a complex procedure by using ProModel's processing element. First of all, to route vehicle from queuing part to toll collection part, we use built-in and user-defined routing rules provide flexibility for modeling all types of routing conditions. If no capacity is available at a series of tollbooth facility, the entity will wait until capacity becomes available at the last tollbooth of a series of tollbooth facilities.

Before routing vehicle to tollbooth facility, vehicle selects a location only if it is completely empty. Tollbooth facility must be completely empty before it is initially selected. If first tollbooth is not empty, vehicles are routed to ALTERNATE locations. To represent this, we use the EMPTY condition designed for the situation where facility are being filled from the queuing source. Once all locations are filled, no more routings occur until one of the locations becomes empty. Thus the output waits until one becomes empty. Tollbooth must be available at that location before the routing actually takes place.

To describe tandem booth except for the first tollbooth, we define it as an alternate destination if it has available capacity and a preceding tollbooth is not available. If the ALTERNATE locations are also unavailable, the entity waits until it becomes available or all series of tollbooth are available. In this case, we assume that first tollbooth facility might be preferred when all of tollbooths are empty, but alternatively, a second toll facility can be used when the first tollbooth is unavailable.

If a preceding driver's service is not finished completely, all the following drivers are blocked independent of whether or not they have been already served completely because any following vehicles cannot pass their preceding vehicle in the tollbooth facilities. This blocking "rule" may cause additional delay time, Hong et al. (2009). To prevent vehicle from entering tollbooth facilities until preceding vehicles complete toll services as blocking rule, operation logic includes control (IF-THEN-ELSE) statements, nested statement blocks and subroutine calls. WAIT UNTIL statement causes a car entity to remain at the current queue location until last one of series of toll

service is complete. Once all tollbooths are empty, following vehicle group can use tollbooth facilities after predecessor group finishes toll service.

To describe shockwave phenomenon, which means that a type of propagating disturbance of flow, on traffic flow, we generate reaction time distribution and assign it to each vehicle. Reaction time generally propagates backward in traffic flow due to shockwave. Congestion in queue stemming from a driver's reaction has an effect on rear vehicles in the tollbooth systems. As the number of tollbooths increases, therefore, reaction effect in a queue becomes larger and, at the same time, simultaneous usability of tollbooth is decreased steeply (Hong et al. 2009). To represent this phenomenon, we use WAIT UNTIL statement to make entities held at a current location until predecessor completes its toll service. After final toll payment approval is received, a vehicle waits more for the amount of reaction time and then is sent to the appropriate toll facilities. After a vehicle completes tollbooth operation at tollbooth facilities, it routes to departure part where in the model terminates entities and collect statistical performance measures.

#### *Step 4: Departure Part*

Normally, front area of toll facility is vacuumed because it is vacuum high-speed area. Therefore, we assume that vehicle which completes toll service are gone out without any delay. In this part, we calculate waiting time in plaza for a basic output of several MOE (Measure of Effectiveness)s. The simulation model mentioned above can be described as an object-oriented component.

## COMPUTATIONAL EXPERIMENT

### **Experimental Design**

Tandem tollbooth simulation model and algorithm described above were implemented in ProModel 7.5, a powerful object-oriented modeling and simulation software, and run on an IBM compatible PC with 2.80 GHz Intel Core2 Duo processors, utilizing the Microsoft Windows 7 operating system.

For implementation of simulation, it is important to decide the number of replications and the simulation run length. To get a sta-

tistical significance, we conducted 300 replications with different random number streams to obtain a mean and variance of throughput capacity with statistical significance. To represent the state of tandem tollbooth situation at peak time, we use 10minute warm-up and 1 hour running time option before collecting statistics. We use the linear congruential generator which is one of the most common Pseudo-random number generators.

Results from these runs were averaged for each replication with 10 minutes warm-up period. Comparisons between model outputs and actual field observations were conducted. Three different MOEs were selected for each lane to validate ProModel-Based tandem toll-booth model: throughput, average queuing delay, and total queuing delay. The MOEs are explained in the following sections.

Performance Measurement

Here, we show how the performance measures normally chosen in the analysis of queuing systems may be represented on the basis of the models described above. We start with representative performance measures associated with the toll plaza systems in the general case, and then briefly discuss the evaluation of system performance measures under consideration.

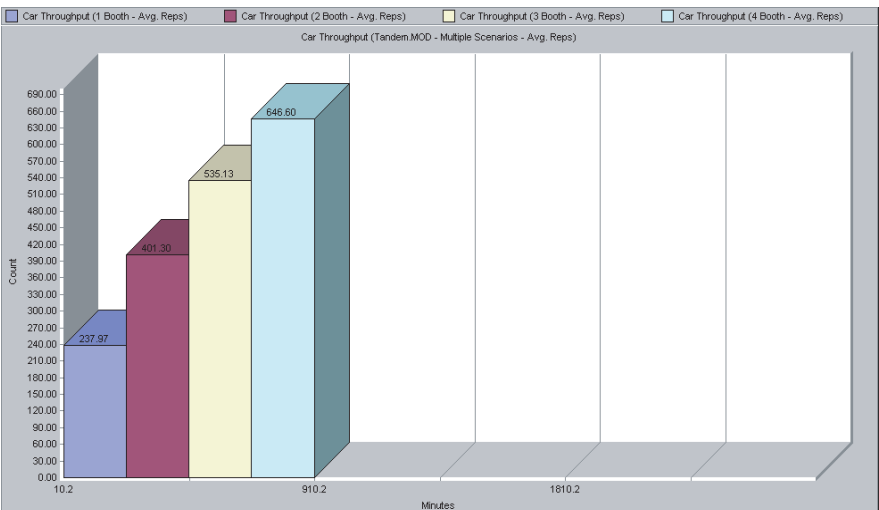


Figure 3. Throughput

Throughput

By running 10-minute warm-up and 1 hour running time option, we estimate the total number of vehicles that exit within 1 hour. If we operate tandem booth, the capacity increase to 401 vehicles per hour compared with the capacity of the single booth, 238 vehicles. It is almost twice the capacity of the single booth. But the effect of additional serial booth on the capacity decreases as the number of tollbooth attached is increases.

Average Simulation Time by Reaction Time

It is assumed that service time approximates normal distribution. By limiting arrivals to 240 vehicles, we compare the result of total time when all the vehicles exit. Reaction time is physiologically defined as the elapsed time between perceiving a stimulus and performing a reaction. The reaction time in traffic flow influences the stability of traffic flow because the elapsed time between stimulus and response creates and propagates disturbance of flow on the road. Therefore, it is needed to analyze the effect of reaction time on throughput of tandem tollbooth system. In this paper, we define the reaction time ratio ( $\theta_r$ ) as the ratio of mean reaction time to mean service time ( $\theta_r = \text{mean reaction time} / \text{mean service time}$ ). In this condition, Figure 4 shows that decreasing reaction time ratio ( $\theta_r$ ) result in decreasing average exit time. In the case that reaction time ratio

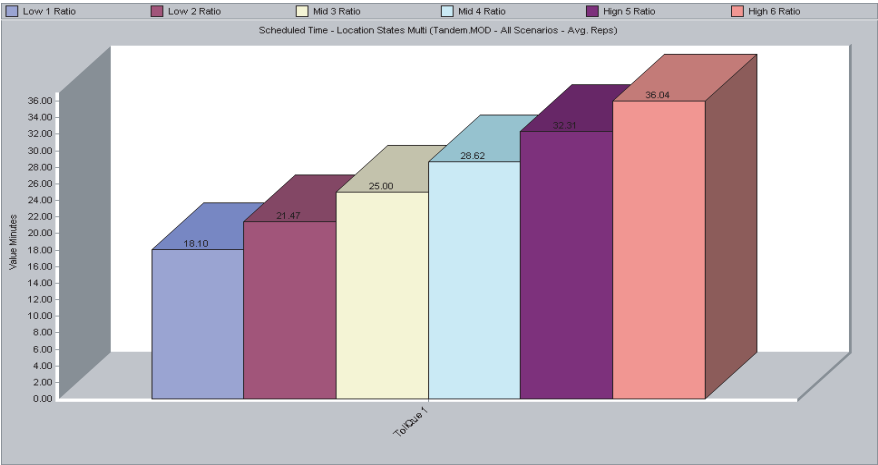


Figure 4. Average Simulation Time by Reaction Time

is 0.1 ( $\theta_{lr} = 0.1$ ), the average exit time is 18.10 minutes, which is half of the exit time in the case that reaction time ratio is 0.6 ( $\theta_{lr} = 0.6$ ). The result shows that the capacity in a tandem tollbooth, therefore, is expected to increase as reaction time ratio ( $\theta_{lr}$ ) decreases.

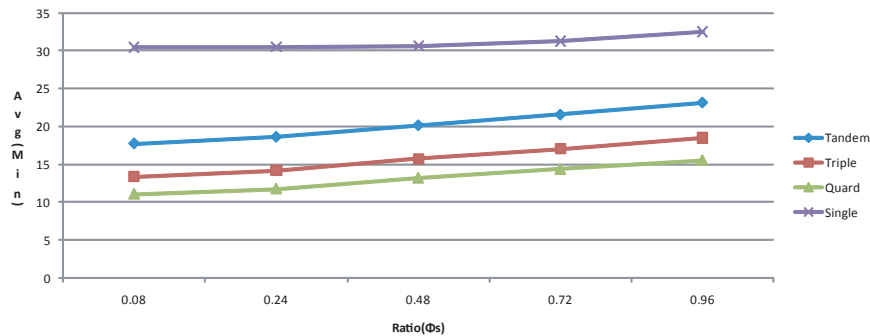
*Average Simulation Time by Service Time*

Figure 5 depicts the change of the capacity according to the coefficient of variation of service time ( $\phi_s = \text{standard deviation of service time} / \text{mean service time}$ ). The result shows the standard deviation of service time has little effect on the total throughput. This means the capacity doesn't change no matter what the standard deviation of service time is. In this case, the capacity depends only on the number of tollbooth facilities.

*Blocking Time by Service Time*

We assume that service time has a normal distribution. Figure 6 shows that the increasing ratio of the standard deviation of service time ( $\phi_s$ ) increases the average blocked time linearly. This means that the higher standard deviation is, the more inconvenience the driver feels.

This result suggests that the standard deviation of service time of tollbooth systems should be controlled to minimize the blocking time by preceding vehicles. Even if overall throughput does not change with the standard deviation of service time, each driver wants to avoid inconvenience and uncomfortable situation caused by blocked time. For service-providers (operators), toll-collectors should be controlled to collect toll payments quickly and uniformly. For service-



**Figure 5. Average Simulation Time by Service Time**

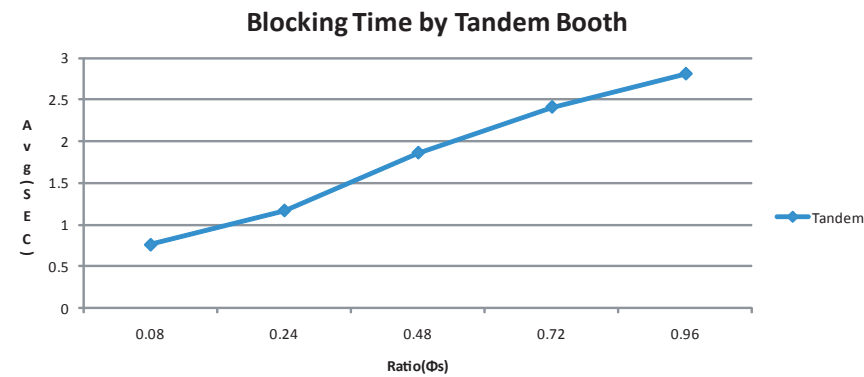


Figure 6. Blocking Time by Service Time

consumers (drivers), vehicles should avoid various payment types which lead to a large variation of service time.

CONCLUSION

This research examines object-oriented approach to analyze tandem tollbooth. Our analysis shows that the capacity of tandem tollbooth varies on the service time of toll takers and reaction time of drivers. A skillful toll collector with a low mean and variance of service time and an identical payment type that guarantees uniform service time is helpful to decrease the blocking time. For example, if a toll lane is dedicated to one payment type, such as touch-and-pay card, it will reduce total delay time by being operated as a tandem tollbooth. Furthermore, because the service time varies on the vehicle type, it is necessary for different vehicles to get toll service separately. Regarding service time, it is recommended to reduce the reaction time of drivers. Certain type of signal systems that make all the vehicles in a queue to react simultaneously could effectively increase the capacity of the toll lane. It is recommended that Tandem tollbooth is installed with a signal system that alerts drivers in advance to reduce blocking time and to make tandem tollbooth used simultaneously.

This research focused on the service parts at a tollbooth without considering the arrival pattern and acceleration/deceleration of vehicles. We assume the same width and height of each vehicle and

the same structure size of tollbooths. In reality, there exists a wide range of size and arrival rate. To reflect real situation, it is recommended to collect detailed real toll data. Finally, further research on the cost/benefit analysis of the tandem toll booth should be made to verify its economic value. In addition, to validate the flexibility of Pro-Model based tandem tollbooth model suggested by this research, it needs to collect detailed vehicle-by-vehicle toll data and physical structure of tollbooth at each toll plaza in reality.

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