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Item Aggregates and Price Elasticity

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Abstract

This study provides analytical results on the systematic relationship between the elasticities obtained from item-aggregated data and those from SKU level data. It is shown that the brand level (or any aggregate level) elasticities are share-weighted averages of SKU elasticities. As SKUs in a brand are substitutes each other in general, the own brand elasticities would be smaller in magnitude than the own SKU elasticities and the cross-brand elasticities will be larger than cross-SKU elasticities. It is also found that when the SKU level demand function is given by a homogeneous logit model with the latent utility being linear in price, the price sensitivity parameter estimated from the brand level data should be the same as that from SKU level data if the brand level model fits data well and the brand prices are given by the weighted averages of SKU prices with the weights being within-brand SKU shares.

Keywords: Item Aggregates, Price Elasticity, Logit Model

INTRODUCTION

Most consumer packaged good categories have hundreds of items called stockkeeping units (SKUs) or UPCs in a category. Such a large number of SKUs in a category posits challenges to marketing researcher in modeling the demands for products in the category. One simple approach would be to limit the analysis to a few selected items with most shares so that the data set has a manageable number of alternatives. Such a data pruning practice can invite

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significant bias in parameter estimates and implied elasticities (Zanutto and Bradlow 2006). Alternatively, marketing researchers have modeled demands for the products in a category mostly at the brand level or at the brand-size level rather than at the SKU level in order to reduce the number of choice alternatives. In brand level or brand-size level studies, aggregate measures such as aggregate prices and aggregate demands are constructed even when SKU level scanner data are available. Typically, the item aggregation would be motivated by practical reasons such as the computational requirements involved in the model estimation process or by the nature of data such as sparse observations at SKU level or volatile choice sets due to the frequent entry and exit of SKUs (Bucklin and Gupta 1999). More importantly, it is also often the case that the item aggregation is required by the nature of the research question being involved in the study. For example, in order to assess the impact of the merger between two brands in a category on the market demand and the equilibrium pricing, one needs to figure out the substitution pattern at the brand level rather than at the SKU level.

The outcomes of brand level studies would include the estimates of own and cross-brand price elasticities. One of the potential issues would be how the estimates obtained from the data of item aggregates can be related to the demand characteristics for disaggregate units, SKUs. It has been empirically observed that item aggregation would have an impact on the estimates of demand responses for marketing activities. For example, the meta analysis by Bijmolt, van Heerde, and Pieters (2005) reports that brand level studies produce significantly smaller price elasticities than SKU level studies, -2.50 for brands vs. -2.97 for SKUs.

In this paper, I investigate the systematic relationship between the brand level price elasticity and the SKU level price elasticity. I try to reconcile the difference between elasticities from different aggregation levels and to provide mathematical relationship between them. I focus on price elasticity for several reasons. First, price elasticity is one of the key pricing issues identified by practitioners (Bucklin and Gupta 1999). Managers recognize that understanding price elasticity is the basic starting point for better pricing. Second, price elasticity is also a quantity of interest to policy makers as it plays a key role in several issues in industrial organization studies such as market structure, merger and acquisition, and so on. Third, as documented well in Tellis (1988) and Bijmolt, van Heerde, and

Pieters (2005), price elasticity has been one of the key research issues among academics. Moreover, although I limit my analysis to price elasticity in this paper, the structure of the analysis can also be applicable directly to the elasticity measures for other marketing activities such as promotion.

Unlike Allenby and Rossi (1991), the aggregation issue I investigate in this paper is the aggregation across product items, not across households or across stores. While relatively little attention has been given to the issue of item aggregation, there is relatively rich literature on the issue of data aggregation across households or across stores. For example, Christen et al. (1997) show that the estimates of promotion effects calibrated from linearly aggregated market level data would be substantially different from those obtained from store level data. Gupta et al. (1996) find that panelist households in household level scanner data are not representative enough to reflect the demand characteristics of store level data from the same community, although the average price elasticity estimates are close for both data sets. Unlike Christen et al. (1997) and Gupta et al. (1996), my interest in this paper is how the price elasticities obtained from item aggregates (brands) can be related to the price elasticities of demands for individual items (SKUs). While any differences in estimation outcomes between aggregate data (market level data) and disaggregate data (store level or household level data) would be considered bias in their studies, I view the difference between the price elasticities from item aggregates data and those from individual items data as natural and try to reconcile the difference.

The issue of item aggregation has been studied in marketing area. The main theme of such studies is how to estimate the SKU level preference parameters. Fader and Hardie (1996) and Ho and Chong (2003) provide ways to analyze household level SKU choices by building a parsimonious model so that the number of parameters would not explode with the number of SKUs considered. These studies would impose a particular set of restrictions on the structure of SKU level demand in order to keep the number of parameters manageable. Bell, Bonfrer, and Chintagunta (2005) utilize store level data and provide a way to recover SKU level preference parameters from the market share model estimated from item aggregates. They exploit the particular structure of the functional form of the demand – logit. Unlike those studies, the focus of my study is not on how to

estimate the SKU level elasticity. The key issue in this paper is how and why the brand level (or any other item aggregate level) elasticity is different from the SKU level elasticity. Without imposing any particular structure or functional form on the SKU level demand, I show how the price elasticity obtained from item aggregates can be related to the price elasticity obtained from individual items so that the findings can be applied to a general set of demand models.

The remainder of the paper is organized as follows: In the next section, an analytical model on the impact of item aggregation on the price elasticity is presented. Then, I explore how the functional form for the demand would be affected by item aggregation. Analytical results along with some simulation results are provided. Finally, a brief conclusion follows.

GENERAL MODEL

Consider a model for SKU level demands in a category. Suppose there are M brands in the category and J_m SKUs in brand m=1,..,M. While I use the subscript m to denote brand, it can be any level of item aggregates such as brand-size or product line. I use a general demand function for SKUs as follows:

$$Q_{mj} = Q_{mj}(p_{11},\ldots,p_{1J_1},p_{21},\ldots,p_{2J_2},\ldots,p_{M1},\ldots,p_{MJ_M}) \tag{1}$$

where Q_{mj} is the demand for SKU j in brand m and p_{mk} is the unit price of SKU k in brand m. Note that the unit price is used in the demand function so that the prices of item aggregates are meaningful. Similarly, the SKU demand is also measured by the common unit such as ounce, not by the number of items sold. I use the following notations to denote the SKU level own and cross price elasticities:

$$\eta^s_{mj,mj} = \frac{\partial Q_{mj}}{\partial p_{mj}}\frac{p_{mj}}{Q_{mj}} \quad \text{and} \quad \eta^s_{mj,nk} = \frac{\partial Q_{mj}}{\partial p_{nk}}\frac{p_{nk}}{Q_{mj}} \tag{2}$$

where the superscript s stands for SKU level elasticity. Now consider the demand for item aggregates (brands), where the brand level data are the linear aggregation of SKU level data,

$$Q_m = Q_m\left(p_1, p_2, \ldots, p_M\right) \tag{3}$$

where Q_m is the demand for brand m and p_m is the price of brand m. The linear aggregation indicates that the brand level data are given as follows:

$$Q_m = \sum_{j=1}^{J_m} Q_{mj} \tag{4}$$

$$p_m = \sum_{j=1}^{J_m} w_{mj} p_{mj},\ w_{mj} \geq 0, \forall m, j,\ and \sum_{j=1}^{J_m} w_{mj} = 1. \tag{5}$$

The brand demand is the sum of SKU demands and the brand price is a weighted average of SKU prices. Note that a different weighting scheme, w_{mj}, for the brand price construction implies a different demand function for (3). Different weights would result in systematically different brand prices while the demand quantities in (4) in the aggregate data are not affected by the weighting scheme. So the parameters and/or even the functional form of the demand function in (3) would be dependent upon the weighting scheme used in the item aggregation process.

In order to derive the brand level price elasticities, I need to clarify the meaning of the change in brand prices. Price elasticity is considered a thought experiment where the percentage change in demand is measured as a consequence of one percent change in price while other things are held constant. So the experiment is meaningful only when researchers compare two points *within* a demand function. Therefore, when researchers change the brand price in the experiment, they should not change the weights, w_{mj}, given to SKU prices. Changes in weights result in a comparison *across* two different demand functions, not *within* a demand function, because the nature of the brand prices in (3) would not be the same. That is, the weights should be the same between two regimes – before price change and after price change. The weight-preserving change in brand price is accomplished by the identical percentage change in prices of all SKUs in the brand. That is, in order to be a legitimate thought experiment, the manipulation, one percentage change in brand price, must be accomplished by one percentage changes in all SKUs. Mathematically, this implies

$$d\log p_m = d\log p_{mj}, \forall j = 1,..,J_m. \tag{6}$$

The change in the demand for brand m due to the change in the price of brand n is given by

$$\begin{aligned} dQ_m &= \sum_{j=1}^{J_m} dQ_{mj} = \sum_{j=1}^{J_m}\sum_{k=1}^{J_n} \frac{\partial Q_{mj}}{\partial p_{nk}} dp_{nk} \\ &= \sum_{j=1}^{J_m}\sum_{k=1}^{J_n} \frac{\partial Q_{mj}}{\partial p_{nk}} \frac{p_{nk}}{Q_{mj}} \frac{Q_{mj}}{p_{nk}} dp_{nk} \\ &= \sum_{j=1}^{J_m}\sum_{k=1}^{J_n} \eta^s_{mj,nk} Q_{mj} d\log p_{nk}. \end{aligned} \tag{7}$$

So the percentage change in brand demand is given by

$$d\log Q_m = \frac{dQ_m}{Q_m} = \sum_{j=1}^{J_m}\sum_{k=1}^{J_n} \eta^s_{mj,nk} \frac{Q_{mj}}{Q_m} d\log p_{nk}. \tag{8}$$

Denote the within-brand SKU shares $\lambda_{mj} \equiv Q_{mj}/Q_m$ and use the weight-preserving condition for brand price changes in (6) to derive the own and cross brand level elasticities as follows:

$$\eta^b_{m,m} = \frac{d\log Q_m}{d\log p_m} = \sum_{j=1}^{J_m} \lambda_{mj} \sum_{k=1}^{J_m} \eta^s_{mj,mk} \tag{9}$$

$$\eta^b_{m,n} = \frac{d\log Q_m}{d\log p_n} = \sum_{j=1}^{J_m} \lambda_{mj} \sum_{k=1}^{J_n} \eta^s_{mj,nk}. \tag{10}$$

where the superscript b indicates the brand level elasticity. The expressions in (9) and (10) show that the brand level elasticity is a weighted sum of SKU level elasticities with the weights given by the within-brand SKU shares. In an extreme case where SKU demands are independent ($\eta^s_{mj,mk} = 0, \forall j \neq k$), the own brand elasticity is nothing but the share-weighted average of own SKU elasticities. But in most real world marketing applications where SKUs within a brand are expected to be substitutes each other in general ($\eta^s_{mj,mk} > 0, j \neq k$), the own brand level elasticities will be smaller than the share-weighted average of own SKU level elasticities in

magnitude. Similarly, the cross-brand elasticity will be larger than cross-SKU elasticity. If the price of 128oz Tide detergent increases, consumers can switch to a different size of Tide detergent or to a same or a different size in other brands. When the prices of all Tide detergent products increase, consumers will switch only to other brands. So the price elasticity would be smaller (in magnitude) for Tide brand than for 128oz Tide. The expressions in (9) and (10) provide an analytical relationship of price elasticities between different levels of aggregation. They imply that the substitutability should be the only factor related to any difference between the brand level elasticity and the SKU level elasticity if the analysis is done for the same demand group (a group of households or a market). In general, a broader definition of a product would result in a smaller own price elasticity for the aggregated item. A longer time frame in a demand model would result in smaller own elasticities. It has been reported empirically that a static demand model ignoring intertemporal substitution would produce larger estimates for own elasticities (Hendel and Nevo 2006) and smaller cross elasticities (Erdem, Imai, and Kean 2003).

The demand structure and the nature of item aggregation also provide information on the relationship among brand level elasticity, SKU level elasticity, and the elasticity of within-brand SKU shares. In order to derive the elasticity of the within-brand SKU shares, first note that the percentage changes in relative shares of SKUs in a brand is given by

$$d\log\lambda_{mj} = d\log\frac{Q_{mj}}{Q_m} = d\log Q_{mj} - d\log Q_m. \tag{11}$$

Combining (11) with (6) yields an expression for the elasticity of within-brand SKU shares as follows:

$$\begin{aligned}\eta^{\lambda}_{mj,n} &\equiv \frac{d\log Q_{mj}}{d\log p_n} - \frac{d\log Q_m}{d\log p_n} = \sum_{k=1}^{J_n}\frac{d\log Q_{mj}}{d\log p_{nk}} - \frac{d\log Q_m}{d\log p_n} \\ &= \sum_{k=1}^{J_n}\eta^{s}_{mj,nk} - \eta^{b}_{m,n}.\end{aligned} \tag{12}$$

where the superscript λ indicates the elasticity of within-brand SKU shares. Rewriting (12) produces an expression for the elasticity of a SKU relative to a brand price as follows:

$$\eta^{s}_{mj,n} \equiv \sum_{k=1}^{J_n} \eta^{s}_{mj,nk} = \eta^{b}_{m,n} + \eta^{\lambda}_{mj,n}. \tag{13}$$

The expression in (13) is intuitive in the sense that the effects of brand-wide price changes of all SKUs in brand n on the demand for a SKU in brand m (in the same or other brand) can be decomposed into two effects: brand switching effect ($\eta^{b}_{m,n}$) and within-brand SKU switching effect ($\eta^{\lambda}_{mj,n}$). Note that although I use the term "switching," the brand switching effect can include primary demand effects such as the changes in overall consumption level while the SKU switching effect refers to pure share-adjusting effect. It can be easily verified that the share-weighted sum of the within-brand SKU switching effects is zero by multiplying λ_{mj} both sides of (12) and summing over *j*.

Equation (9) and (10) provide a way to compute brand level elasticities from SKU level elasticities. However, in general it is not feasible to recover SKU level elasticities from brand level elasticities using (9) and (10) unless researchers impose a particular set of restrictions on the structure of SKU level demand as in Fader and Hardie (1996), Ho and Chong (2003), or in Bell, Bonfrer, and Chintagunta (2005).

ITEM AGGREGATES AND FUNCTIONAL FORMS FOR DEMANDS

Another issue related to item aggregation is the functional form for the demand functions in (1) and (3). In general, the functional form does not remain the same as items are aggregated. Consider, for example, linear demand functions for SKU demands. Suppose there is only one brand with 2 SKUs in the market. The SKU level linear demand is given by

$$Q_1 = \alpha^{s}_{1} + \beta^{s}_{11} p_1 + \beta^{s}_{12} p_2 + \varepsilon_1, \text{ and } Q_2 = \alpha^{s}_{2} + \beta^{s}_{21} p_1 + \beta^{s}_{22} p_2 + \varepsilon_2. \tag{14}$$

So the "true" demand function for the item aggregate (brand) is

$$Q = Q_1 + Q_2 = \alpha^{s}_{1} + \alpha^{s}_{2} + (\beta^{s}_{11} + \beta^{s}_{21}) p_1 + (\beta^{s}_{12} + \beta^{s}_{22}) p_2 + \varepsilon_1 + \varepsilon_2. \tag{15}$$

Is the demand function in (15) is linear in "brand" price? The (possibly ill-specified) linear demand function for the brand demand based on aggregate price would have the following form:

$$Q = \alpha^b + \beta^b p + \varepsilon^b = \alpha^b + \beta^b w_1 p_1 + \beta^b w_2 p_2 + \varepsilon^b. \tag{16}$$

Because the brand price is given by $p = w_1 p_1 + w_2 p_2$ from (5). The two linear demand functions, (15) and (16), are equivalent only when

$$\beta^b = \frac{\beta_{11}^s + \beta_{21}^s}{w_1} = \frac{\beta_{12}^s + \beta_{22}^s}{w_2}. \tag{17}$$

That implies that if the weights used to construct aggregate price do not satisfy the condition $w_1 / w_2 = (\beta_{11}^s + \beta_{21}^s)/(\beta_{12}^s + \beta_{22}^s)$ then there does not exist a linear function for brand demand when the true SKU level demand is characterized by a linear function. The situation gets worse when it comes to nonlinear demand functions such as the multiplicative demand function or the log-log function, which is frequently used to estimate price elasticity in many marketing applications. Consider the following log-log demand function for the SKUs,

$$\begin{aligned} \log Q_1 &= \alpha_1^s + \beta_{11}^s \log p_1 + \beta_{12}^s \log p_2 + \varepsilon_1, \\ \log Q_2 &= \alpha_2^s + \beta_{12}^s \log p_1 + \beta_{22}^s \log p_2 + \varepsilon_2. \end{aligned} \tag{18}$$

If (18) is the true model, then the true aggregate demand is given by

$$\begin{aligned} Q = Q_1 + Q_2 &= \exp(\alpha_1^s + \beta_{11}^s \log p_1 + \beta_{12}^s \log p_2 + \varepsilon_1) \\ &\quad + \exp(\alpha_2^s + \beta_{21}^s \log p_1 + \beta_{22}^s \log p_2 + \varepsilon_1). \end{aligned} \tag{19}$$

The demand in (19) cannot be expressed as $\log Q = \alpha^b + \beta^b \log (w_1 p_1 + w_2 p_2) + \varepsilon^b$ regardless of the weights. It is expected that when the "true" demands for SKUs are given by a nonlinear demand function the brand level demand function will not follow the same functional forms. Note that it is possible that there exits a correct functional forms for brand level demand which is likely to be different from the functional form for SKU level demands. While

this issue is beyond the scope of this study, interested readers are directed to Deaton and Muellbauer (1980) Chapter 5.

What would happen if researchers impose the same functional forms for demands at different levels of aggregation? While such issue has been studied little for most nonlinear functions, one notable exception is the logit model. As the extreme value distribution is maintained under maximization (i.e., the maximum of independent extreme value distributed random variables is also extreme value distributed), if the SKU level market share follows a logit model, the brand level market share is also characterized by a logit model. However, one cannot say it has the same functional form. In fact, the true utility structure is no longer linear in the brand price index in the model for the item aggregates even if the true utilities in the SKU level model are linear in SKU prices. It is analogous to the nested logit structure. (See Chapter 9 of Ben-Akiva and Lerman (1985) for a detailed discussion.) Interestingly, I find that imposing the same linear-in-price structure on the utility in the model for item aggregates would impose a restriction on the price sensitivity parameter. Consider a SKU level logit model where the market share of the SKU j of brand m (s_{mj}) is given by

$$s_{mj} = \frac{\exp(\alpha_{mj}^{s} + \beta^{s} p_{mj})}{\sum_{n=1}^{M}\sum_{k=1}^{J_n}\exp(\alpha_{nk}^{s} + \beta^{s} p_{nk})}. \tag{18}$$

The true brand level share is obtained by summing the shares of all SKUs. The brand share (s_m) and the within-brand SKU shares (λ_{mj}) are given as follows:

$$s_m = \sum_{j=1}^{J_m} s_{mj} = \frac{\sum_{j=1}^{J_m}\exp(\alpha_{mj}^{s} + \beta^{s} p_{mj})}{\sum_{n=1}^{M}\sum_{k=1}^{J_n}\exp(\alpha_{nk}^{s} + \beta^{s} p_{nk})}, \text{ and}$$
$$\lambda_{mj} = \frac{s_{mj}}{s_m} = \frac{\exp(\alpha_{mj}^{s} + \beta^{s} p_{mj})}{\sum_{k=1}^{J_n}\exp(\alpha_{mk}^{s} + \beta^{s} p_{mk})} \tag{19}$$

Using the result in (9), it can be easily shown that the true own brand elasticity for the demand in (19) is given by

$$\eta^{b}_{m,m} = \beta^{s}\left(\sum_{j=1}^{J_m} \lambda_{mj} p_{mj}\right)\left(1 - s_m\right) \tag{20}$$

Suppose the following linear-in-price structure is used to model brand level market shares

$$s^{a}_{m} = \frac{\exp(\alpha^{a}_{m} + \beta^{a} p_m)}{\sum_{n=1}^{M} \exp(\alpha^{a}_{n} + \beta^{a} p_n)} \tag{21}$$

where the superscript a indicates the aggregate level model which is possibly misspecified. The model in (21) would produce the following expression for own brand price elasticity:

$$\eta^{a}_{m,m} = \beta^{a} p_m \left(1 - s^{a}_{m}\right) = \beta^{a}\left(\sum_{j=1}^{J_m} w_{mj} p_{mj}\right)\left(1 - s^{a}_{m}\right). \tag{22}$$

If the model in (21) fits the data well ($s^{a}_{m} \cong s_m$) and the weights used to construct brand price indices are equal or close to within-brand SKU shares ($w_{mj} \cong \lambda_{mj}$), then the price sensitivity parameter in (21) should be close to that in (18), $\beta^{s} \cong \beta^{a}$. I conduct a small scale simulation to verify it. In the simulation, there are 2 brands with 2 SKUs in each brand. The SKU level utility is given by $U_{mjt} = \alpha_{mj} + \beta p_{mjt} + \varepsilon_{mjt}$ where the true parameters are α = {0.5, 0.6, 0.7, 0.8} and β = -2. SKU prices are generated by adding independent uniform random numbers, u(0, 1), to the mean prices of 4 SKUs, {0.5, 0.7, 0.8, 1.0}. I generate 10000 observations. I estimate the SKU level logit model in (18). And then I aggregate data into brand level and estimate the brand level logit model in (21) using the brand level data. The aggregate prices are weighted average of SKU prices where weights are within-brand SKU shares computed across all observations. I repeat the simulation 50 times. As not all product intercepts are separately identified in the logit model, I normalize the intercept for the first product to zero. Note that the SKU level model is the true model used to generate the data. The simulation results in table 1 show that the brand level model produces almost identical results as the SKU level model does. Although Ben-Akiva and Lerman (1985: 259) also expect

Table 1. Simulation Results with Same Number of SKUs

	SKU Level Model (β^s)	Brand Level Model(β^a)
Average Estimates across replications	-2.0070	-1.9959
Std. Dev. across simulation replications	0.0525	0.0893
Average Standard Error	0.0399	0.0715
Average Difference (β^s- β^a)	-0.0111	
Std. Dev. of Difference (β^s- β^a)	0.0738	

Table 2. Simulation Results with Different Number of SKUs

	SKU Level Model (β^s)	Brand Level Model(β^a)
Average Estimates across replications	-1.9948	-2.0023
Std. Dev. across simulation replications	0.0483	0.0833
Average Standard Error	0.0405	0.0859
Average Difference (β^s- β^a)	0.0075	
Std. Dev. of Difference (β^s- β^a)	0.0730	

such results when the size and the variance measures are adjusted properly, I do not control for such factors in the brand level model in the simulation. I do not include the variance measure in the brand level model so that the brand level model is misspecified. However, the size and the variance measure are quite similar across brands, so they may be cancelled out in the simulation model.

I conduct another set of simulation where the number of SKUs is different across brands. In this set of simulation, brand 1 has 3 SKUs and brand 2 has only one SKU. Under this case, the size and the variance are different across brands so they would not be cancelled out. I keep the other simulation parameters the same as before. The results in table 2 indicate that the estimate of price sensitivity parameter from the aggregate data is again almost identical to that obtained from SKU level data, which verifies well the relationship in (20) and (22).

The analytical results in (20) and (22) together with the simulation results indicate that, as long as the brand level logit model fits data well, the price sensitivity parameter obtained from the brand level data will be the same as (close to) the price sensitivity parameters

from the SKU level data in a logit model. Although there is little literature that empirically investigates the impact of item aggregation on the estimate of price sensitivity parameter, Bell, Bonfrer, and Chintagunta (2005) provides some results on the issue. However, they report findings inconsistent with my results. They found that the price sensitivity calibrated from SKU level toothpaste data is -5.47 whereas the price sensitivity estimates from brand level data is -6.18. While it is not a key research issue in their paper, they also report the estimates of price sensitivity obtained from aggregate data based on various aggregation schemes such as flavor, form, functions, and size. Their price sensitivity estimates dramatically differ along the dimension used to aggregate items. For example, when the data were aggregated into flavor level (i.e., SKUs with same flavor are aggregated into a choice alternative), the price sensitivity estimate was -8.74. Such finding is inconsistent with the property of the homogeneous logit model. Specifically, "if the average utilities are well specified, …, the parameters of the choice model are not dependent on the definitions of the aggregate alternatives." (Ben-Akiva and Lerman 1985: 259) As they use market-share weighted average prices for the aggregate prices, their results might be due to either (1) relatively poor data fit at the aggregate model or (2) the possibility that the homogeneous logit model is not the true model underlying SKU level data generating process, unlike the assumption in my simulation.

In order to check the impact of the model misspecification at the SKU level, I conduct one more set of simulation where the error terms in the utility function follow normal distributions while other simulation parameters are the same as in table 2. To make the model very different from logit, I assume that error terms are heteroscedastic but still independent across choice alternatives. The standard deviations of the error terms are 1, 3, 4, and 6 for 4 SKUs respectively. As expected, the results in table 3 show that the estimates are far from the true value. In addition, the estimate of price sensitivity obtained from the SKU level data is significantly different from that from brand level data. It implies that if the homogeneous logit is not the true data generating process for SKU level data, the estimates obtained from item aggregates would be different from those from SKUs. It also implies that comparing price sensitivity estimates obtained from different aggregation levels might be a indirect way to check whether the homogeneous logit model is

Table 3. Simulation Results with Misspecified SKU Level Model

	SKU Level Model (β^s)	Brand Level Model(β^a)
Average Estimates across replications	-0.7877	-0.5042
Std. Dev. across simulation replications	0.0366	0.0662
Average Standard Error	0.0355	0.0641
Average Difference (β^s- β^a)	-0.2835	
Std. Dev. of Difference (β^s- β^a)	0.0544	

Table 4. Simulation Results with Larger Price Variance

	SKU Level Model (β^s)	Brand Level Model(β^a)
Average Estimates across replications	-1.9984	-1.967
Std. Dev. across simulation replications	0.0374	0.0559
Average Standard Error	0.0318	0.053
Average Difference (β^s- β^a)	-0.0313	
Std. Dev. of Difference (β^s- β^a)	0.0551	

far from the true model.

What would happen if prices become more volatile? In order to investigate if the qualitative implications would remain the same even when price variances are large, I conduct another simulation where the price is randomly drawn from u(0,1.5) while keeping other simulation settings the same as in table 1. As presented in table 2, the new simulation produces a similar result as the first simulation shown in table 1. That is, the brand level model produces almost identical results as the SKU level model does even when price variance is larger.

One interesting issue would be to check what difference will be observed between SKU level estimates and brand level estimates from real data. I conduct a small scale empirical analysis using a real data set from a panel of consumers who purchased ground coffee products at a store in Chicago area. The data set consists of 203 purchase observations from 69 consumers over 62 weeks starting from June 1991. As presented in table 5, I selected seven major SKUs from top three brands – Folgers, Hills Brothers, and Maxwell House. According to the data set, Hills Brothers has the

Table 5. Estimation Results from Coffee Data

Brand	SKU	Descriptive Statistics						Estimates			
		SKU Choice Share	SKU Unit Price (per oz)		Brand Choice Share	Brand Unit Price (per oz)		SKU Level Model		Brand Level Model	
			Mean	Std. Dev.		Mean	Std. Dev.	Est.	S.E.	Est.	S.E.
Folgers	Folgers 29oz	0.0246	13.44	0.81	0.2463	14.21	1.56	0	Fixed	0	Fixed
	Folgers 26oz	0.2217	14.29	1.71				2.7754	0.4640		
Hills Brothers	Hills Brothers 26oz	0.1773	13.48	2.64	0.5616	13.20	1.47	1.9873	0.4677	0.3189	0.2042
	Hills Brothers 24oz	0.0542	15.69	2.71				1.7962	0.5507		
	Hills Brothers 39oz	0.3300	12.65	1.66				2.2218	0.4432		
Maxwell House	Maxwell House 26oz	0.1379	14.72	1.05	0.1921	15.05	1.07	2.6067	0.4839	0.3520	0.2397
	Maxwell House 24oz	0.0542	15.91	1.13				2.3843	0.5587		
Price Coefficient								-0.6257	0.0607	-0.7355	0.0910

largest relative share among the three. Within a brand, SKUs are different only in their package sizes. SKU level choice shares vary across SKUs even within a brand. The price variation among different package sizes within a brand indicates that firm pricing behaviors are consistent with volume discounting. In terms of the brand level prices, Hills Brothers is the cheapest among the three while Maxwell House is relatively expensive.

I estimate a logit brand choice model where the utility specification is the same as in the series of simulations. That is, the utility of a product consists of the product specific dummy and the price effect. Given such utility specification, I need to estimate product specific intrinsic preferences and a price coefficient. Given the conditional choice specification, I need to normalize the intrinsic preference parameter for a product to zero. All the product specific intrinsic preference parameters should be interpreted as the relative preference over the normalized product. For the SKU level model, I normalize the intrinsic preference for Folgers 29oz to zero. For the brand level, the normalized brand is Folgers.

All of the intrinsic brand preference parameters are significant in the SKU level estimation results. That is, all the other SKUs are preferred over Folgers 29oz. It is intuitive as this product has a smallest share even though it is relatively cheap. On the contrary, none of the intrinsic preference parameter estimate is significant in the brand level model. So the data set does not provide any evidence that Hills Brothers or Maxwell House is preferred over Folgers. Next, what happens to the price coefficient? Although the estimates of the price coefficients look different between the two models, the 95% confidence interval of the SKU level estimate includes the brand level estimate. Also the 95% confidence interval of the brand level estimate includes the SKU level estimate. This result, combined with the implication of the simulation results presented in table 3, might imply that the homogeneous logit model is a good candidate to describe the underlying data generating process for the coffee data used in the analysis.

CONCLUSION

I show how the elasticities of item aggregates are related to the elasticites for individual items. I find that that the price elasticities

for item aggregates are share weighted averages of the elasticities for disaggregate units, indicating the substitutability is the only factor related to the difference between elasticities for different aggregations. As SKUs are substitutes in general, the own brand elasticity will be smaller than the own SKU elasticity and the cross-brand elasticity will be larger than cross-SKU elasticity. The difference between brand elasticity and SKU elasticity is by no means a bias. It is a pure effect of within-brand SKU substitutability. While in general the functional form of demand function would not be preserved as items are aggregated, the homogeneous logit model turns out to be robust to such item aggregation and price sensitivity parameter estimate is not affected by item aggregation as long as the logit model is the true model underlying the SKU level data generating process.

Although this paper is mainly focused on methodological issue, it has managerial implications. Most of all, this paper provides a theoretical ground on why managers should take into account the cross price elasticities when setting base prices. Economic theories suggest that it is optimal for a monopoly firm to set the price at a level equal to the inverse of own price elasticity. If such principle is blindly applied as a rule of thumb to all SKUs produced by the monopoly firm without taking the cross price elasticities among SKUs into account, the firm will end up underpricing SKUs. It is essential to measure inter-SKU substitutability within a brand in order to optimize pricing. In addition to the base pricing issue, this paper also provides an analytical framework to assess the price implication of a merger between firms. As the merged firm would optimize its price level by taking into account the cross price elasticities, one can compute the optimal post-merger price even before the merger happens. While such analyses have been done in empirical settings in literature, this paper provides an analytical framework behind such analyses.

Although this paper does not try to provide a way to recover SKU level elasticity from brand level elasticity, it would be an interesting venue for future research. The ability to recover easily SKU elasticity from item aggregates would not only make it easy to analyze the demand for thousands of SKUs but also enable researchers to infer the pricing behaviors in a micro setting. Although the issue has been studied by Bell, Bonfrer, and Chintagunta (2005), it would be interesting to study how to recover SKU elasticities from item

aggregates with a more general demand function.

Another important issue that is not explicitly analyzed here is the possibility of missing information. If some of SKUs are not included in the analysis, such omission can create bias. Unlike in analyses of simulated data, researchers cannot have all possible SKUs in their empirical analyses of real data. Zanutto and Bradlow (2006) show that pruning the data to a manageable number of SKUs can create bias especially when model fit is poor, when random utility errors are correlated with covariates, or when the model is misspecified. While simulation results in this paper are free from such bias, empirical analyses are subject to such bias. The interaction between data pruning and item aggregation would be an interesting venue for future research.

REFERENCES

Alleby, G. M. and P. E. Rossi (1991), "There Is No Aggregation Bias: Why Macro Logit Models Work," *Journal of Business and Economic Statistics*, 9(January), 1-11.

Ben-Akiva, M. and S. R. Lerman (1985), *Discrete Choice Analysis: Theory and Application to Travel Demand*, MIT Press, Cambridge, MA.

Bell, D. R., A. Bonfrer, and P. K. Chintagunta (2005), "Recovering Stockkeeping–Unit-Level Preferences and Response Sensitivities from Market Share Models Estimated on Item Aggregates," *Journal of Marketing Research*, 42(May), 169-182.

Bijmolt, T. H. A., H. J. Van Heerde, and R. G. M. Pieters (2005), "New Empirical Generalizations on the Determinants of Price Elasticity," *Journal of Marketing Research*, 42(May), 141-156.

Bucklin, R. E. and S. Gupta (1999), "Commercial Use of UPC Scanner Data: Industry and Academic Perspectives," *Marketing Science*, 18(3), 247-273.

Christen, M., S. Gupta, J. C. Porter, R. S., and D. R. Wittink (1997), "Using Market-Level Data to Understand Promotion Effects in a Nonlinear Model," *Journal of Marketing Research*, 34(August), 322-334.

Deaton, A. and J. Muellbauer (1980), *Economics and Consumer Behavior*, Cambridge University Press.

Erdem, T., S. Imai, and M. P. Keane (2003), "Brand and Quanitity Choice Dynamics Under Price Uncertainty," *Quantitative Marketing and Economics*, 1(1), 5-64.

Fader, P. S. and B. G. S. Hardie (1996), "Modeling Consumer Choices among SKUs," *Journal of Marketing Research*, 33(November), 442-452.

Gupta, S., P. Chintagunta, A. Kaul, and D. R. Wittink (1996), "Do Household Scanner Data Provide Representative Inferences From Brand Choices: A Comparison With Store Data," *Journal of Marketing Research*, 33(November), 383-398.

Hendel, I. and A. Nevo (2006), "Measuring the Implications of Sales and Consumer Inventory Behavior," *Econometrica*, 74(6), 1637-1673.

Ho, T. and J. Chong (2003), "A Parsimonious Model of Stockkeeping-Unit Choice," *Journal of Marketing Research*, 40(August), 351-365.

Tellis, G. J. (1988), "The Price Elasticity of Selective Demand: A Meta-Analysis of Econometric Models of Sales," *Journal of Marketing Research*, 25(Novemvber), 331-341.

Zanutto, E. L. and E. T. Bradlow (2006), "Data Pruning in Consumer Choice Models," *Quantitative Marketing and Economics*, 4(3), 267-287.

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