

The Overall t -test Investigating a Conflict between the Individual t -test and the Overall F -test in Regression Analysis^{*}

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ABSTRACT

Researchers may confront conflicting conclusions on the effects of predictors by the individual t -test and the overall F -test in regression analysis. For example, the overall effect may be significant by the overall F -test whereas none of the individual effects are significant by the individual t -test. This paper shows that the conflict may result from different views on the recovered effects of predictors. It proposes an overall t -test assessing the overall effect, and recommends one to use the overall t -test to investigate the conflict between the individual t -test and the overall F -test. The overall F -test assesses the overall effect under the assumption that the true effects of predictors are exactly captured by the recovered effects of predictors. In contrast, the overall t -test assesses the overall effect under the assumption that the true effects are captured by the means and variance-covariances of the recovered effects. This paper ends with practical guidelines for interpreting the effects of predictors when there exists a conflict between the individual t -test and the overall F -test.

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INTRODUCTION

Regression analysis is used to uncover general laws and principles governing the behavior under investigation (Lamberts 2000; Rubin et al. 1999; Usher and McClelland 2001). It is one of popular statistical analyses used in various social sciences. For example, the number of papers using regression analysis in a year was around 900 in twelve clinical psychology journals (Ernst and Albers 2017). In addition, regression analysis is used as an important method for testing hypotheses in various areas of business research (Aguinis and Beaty 2005). It is also notable that the statistical principle of regression analysis is used in mediation analysis as well as moderation analysis (e.g., Aiken and West 1991; Cronbach 1987; Hayes 2013; Park and Yi 2023; Sharma, Durand, and Gur-Arie 1981).

In regression analysis, researchers examine the individual effects of predictors with a *t*-test (referred to as the individual *t*-test in this paper) and evaluate the joint effect of predictors with a *F*-test (referred to as the overall *F*-test in this paper). The overall *F*-test examines whether the model is significant; i.e., the null hypothesis is that all regression coefficients are zero. On the other hand, the individual *t*-test investigates whether the effect of a certain predictor is significant independently from the effects of other predictors; i.e., the null hypothesis is that the regression coefficient of the predictor is zero. It is common to diagnose the effects of predictors in a regression model with the two tests. Any statistical package basically provides the results of the two tests.

If *none* of the individual effects are significant, one may expect that the overall effect would not be significant. However, one occasionally confronts a case that the overall effect is significant by the overall *F*-test, whereas *none* of the individual effects are significant by the individual *t*-test. There may also exist a case that the overall effect is not significant by the overall *F*-test, whereas individual effects are significant by the individual *t*-test. In such cases, researchers may experience the conflict between the overall *F*-test and the individual

t -test. It is possible to confirm that many researchers have a question on the conflict with Web search engines.

One may argue that the conflict between the overall F -test and the individual t -test is not inconsistent with statistical thinking, because such conflict can result from the correlations among predictors. This interpretation on the conflict is reasonable and commonly used in the literature. Correlations among predictors are reflected in the overall F -test, but not in the individual t -test. When predictors are highly correlated, the overall F -test may suggest that the predictors are useful at least when they are used simultaneously, whereas the individual t -test may yield an insignificant result for each predictor, suggesting that *none* of the predictors are needed.

However, the conflict may result from different statistical principles used in the overall F -test and the individual t -test. Let us consider an overall effect of predictors indicating the weighted summed effect of predictors. If there truly exists an overall effect of predictors, the specific overall effect should be significant at least at a set of values of predictors. However, there exists a case in which the specific overall effect is insignificant at all possible sets of values of predictors although the overall effect is significant by the overall F -test. This case implies that the overall F -test cannot reject the null hypothesis on the overall effect although the null hypothesis may be rejected at all possible values of predictors. We will provide an example in the section for a simulation study.

This paper assesses the overall effect with the overall effect magnitude of predictors at possible values of predictors. It specifies the overall effect magnitude with the mean and the variance of a normal distribution. The mean and the variance become zero if all the individual effects of predictors are zero. Thus, this paper insists that it is possible to examine the overall effect with the mean and the variance of the distribution. More specifically, one can assess whether the variance is zero with the overall F -test used in the literature. In addition, one can assess whether the mean is zero with the overall t -test proposed in this paper.

The overall F -test assesses the overall effect with the recovered effects of predictors as the fixed representative measures for the true effects of predictors. In contrast, the overall t -test assesses the overall effect with the recovered effects as a random effect that can be specified with the mean and the variance of the recovered effects. We use the terms of “random” and “fixed” in view that the overall

t -test operates on a random variable and the F -test operates on a constant value.

This implies that the overall F -test and the overall t -test assess the overall effect with different views on the recovered effect of predictors. Accordingly, when there is a conflict between the individual t -test and the overall F -test, this paper recommends one to use the overall t -test to examine the conflict. The overall t -test is consistent with the individual t -test in that the two tests evaluate the effects of predictors with the distributions of the effects. On the other hand, the overall t -test is consistent with the overall F -test in that the two tests evaluate the overall effect of predictors as a whole.

The remainder of the paper is organized as follows. We start with a brief review on the statistical tests in regression analysis. Then, we present the overall t -test and analytically compare it with the overall F -test. Subsequently, we empirically show that the statistical conclusions by the overall t -test may be different from those by the overall F -test. Finally, we provide some implications on the statistical tests in regression analysis.

A BRIEF REVIEW

Overall F -test and Individual t -test

The overall F -test and the individual t -test are well documented in any statistical textbook, because they have been used in various academic fields for more than 100 years. Accordingly, we briefly explain the basic statistical principle used in the overall F -test in a regression model. This basic statistical principle is also adopted in the overall t -test.

A regression analysis diagnoses the individual effects of predictors under the assumption that it is possible to isolate the effect of a predictor from the effects of other predictors. However, when predictors are substantially correlated, it is not possible to assess the individual effects of predictors because changes in one predictor are associated with shifts in another predictor. This phenomenon is referred to as multicollinearity. Multicollinearity affects regression coefficients and corresponding p -values. Thus, the individual t -test is not reliable in assessing individual effects of predictors when there is multicollinearity. Nevertheless, multicollinearity does not

affect the prediction and goodness-of-fit statistics used in the *F*-test. Thus, the overall *F*-test can be used reliably in assessing the overall effect even when there is multicollinearity.

There may be situations in which it is not necessary to examine the individual effects of predictors because it is sufficient to examine the overall effect. On the other hand, there may be situations in which one should examine the individual effects of predictors even when there is multicollinearity because predictors are substantially correlated in nature. For example, it is common to analyze moderation effects with a moderated regression model, which may suffer from multicollinearity by including the interaction term between a focal predictor and a moderator (Echambadi and Hess 2007).

We start with a simple moderated regression model that is well documented in the literature (e.g., Aiken and West 1991; Cronbach 1987; Sharma, Durand, and Gur-Arie 1981). This model can be extended to a more complex model or reduced to a simpler model without the interaction effect. Furthermore, it is possible to compare various tests for the effects of predictors when there is multicollinearity. The moderated regression model is written as:

$$y_i = \alpha + \beta x_i + \gamma k_i + \delta x_i k_i + \varepsilon_i \quad \text{for } i = 1, 2, \dots, N, \quad (1)$$

where x_i and k_i indicate two predictors, y_i indicates a final outcome variable, ε_i indicates an error term [i.e., $\varepsilon_i \sim N(0, \sigma^2)$] for $i = 1, 2, \dots, N$, and N indicates the number of observations. There exists a debate on whether mean-centering can alleviate multicollinearity problems in moderated regression models (e.g., Aiken and West 1991: 182; Echambadi and Hess 2007; Hayes 2013: 283-290). Thus, we start with the moderated regression model that is not transformed with the means.

To evaluate the overall effect of predictors, previous research has used the overall *F*-test diagnosing whether the fit of a regression model (as an unrestricted model) is significantly improved compared to that of the intercept-only model (as a restricted model). In contrast, previous research has used the *t*-test diagnosing whether the individual effects of predictors are zero. In a simple regression model, the *F*-statistic in the overall *F*-test is the squared value of the *t*-statistic in the individual *t*-test. However, the relationship between the two tests in the simple regression model does mean that the two

tests examine the effects of predictors with the same view.

The results by the overall F -test may be inconsistent with those by the individual t -test. The conflicting conclusions may result from the different statistical principles used in the two tests. The overall F -test assesses the overall effect with the estimated effects of predictors as the true effects of predictors. In contrast, the overall t -test assesses the estimated effect of a predictor as a random effect that can be specified with the mean and the variance of the normal distribution.

A potential conflict between the individual t -test and the overall F -test is a well-known problem and there was an attempt to solve the problem. For example, Wilcox (2008) proposes post-hoc analyses that improve the probability of correctly identifying which slope parameters are different from zero based on prediction error. The post-hoc analyses accept the statistical implications from the overall F -test for the overall effect but propose alternative tests for individual effects. Unlike such a strategy, we accept the statistical implications from both the overall F -test and the individual t -tests, and present the overall t -test as an alternative test to investigate a conflict between the individual t -test and the overall F -test.

The Statistical Principle Used in the Overall F -test

The statistical principle used in the overall t -test (that we will present in this paper) is linked with that used in the overall F -test. Thus, we summarize the statistical principle used in the overall F -test in this section. One can express the null hypothesis on the overall effect of predictors as:

$$\beta = 0, \beta = 0, \text{ and } \delta = 0, \quad (2)$$

in the regression model (equation 1). To diagnose the null hypothesis, one should quantify the degree to which the null hypothesis is not empirically satisfied, and develop a statistical test.

The overall F -test diagnoses the null hypothesis on the overall effect with SSE and SSR indicating the sums of squares due to regression of the unrestricted model and the restricted model: i.e.,

$$\text{SSE}_u = \sum_{i=1}^N (y_i - \hat{y}_{ui})^2, \text{ SSR}_u = \sum_{i=1}^N (\hat{y}_{ui} - \bar{y})^2, \text{ and } \text{SST}_u = \sum_{i=1}^N (y_i - \bar{y})^2, \quad (3)$$

$$\text{SSE}_r = \sum_{i=1}^N (y_i - \hat{y}_{ri})^2, \text{ SSR}_r = \sum_{i=1}^N (\hat{y}_{ri} - \bar{y})^2, \text{ and } \text{SST}_r = \sum_{i=1}^N (y_i - \bar{y})^2,$$

where $\hat{y}_{ui}$ indicates the predicted regression model, $\bar{y}$ indicates the mean of the dependent variable, $SST_u = SSR_u + SSE_u$, $SST_r = SSR_r + SSE_r$, and SST , SSR , SSE indicate Sum of Squared Total, Sum of Squares of Regression, and Sum of Squared Errors, respectively.

Let df_u and df_r indicate the degrees of freedom corresponding to SSE_u and SSE_r , and N and m indicate the number of observations and the number of predictors. Then, the *F*-statistic examining the difference between the two models is calculated with:

$$F = \frac{(SSE_r - SSE_u) / (df_r - df_u)}{SSE_u / df_u} = \frac{SSR_u / m}{SSE_u / (N - m - 1)} \sim F(m, N - m - 1), \quad (4)$$

where $df_u = N - m - 1$ and $df_r = N - 1$.

The *F*-statistic is zero when all effects of predictors are zero. It is maximized when the dependent variable is perfectly predicted by predictors. Note that the *F*-statistic is linked with the overall predictive power of predictors because it is represented as a function of R^2 value: i.e.,

$$F = \left[R^2 / (1 - R^2) \right] \left[(N - m - 1) / m \right]. \quad (5)$$

Therefore, the overall *F*-test examines the overall predictive power of predictors.

The SSR_u in the overall *F*-test can be expressed as:

$$SSR_u = \sum_{i=1}^N (\hat{y}_{ui} - \bar{y})^2 = \sum_{i=1}^N \left(\hat{\beta}x_i + \hat{\gamma}k_i + \hat{\delta}x_i k_i - (\hat{\beta}\bar{x} + \hat{\gamma}\bar{k} + \hat{\delta}\bar{x}\bar{k}) \right)^2, \quad (6)$$

where $\hat{y}_{ui} = \hat{\alpha} + \hat{\beta}x_i + \hat{\gamma}k_i + \hat{\delta}x_i k_i$ and $\bar{y} = \hat{\alpha} + \hat{\beta}\bar{x} + \hat{\gamma}\bar{k} + \hat{\delta}\bar{x}\bar{k}$.

It can be simplified to:

$$SSR_u = \sum_{i=1}^N (\hat{y}_{ui} - \bar{y})^2 = (N - m - 1) \text{Var} \left(\widehat{WSEP}(x_i, k_i) \right), \quad (7)$$

where $\widehat{WSEP}(x_i, k_i) = \hat{\beta}x_i + \hat{\gamma}k_i + \hat{\delta}x_i k_i$,

further where $WSEP(x_i, k_i) = \beta x_i + \gamma k_i + \delta x_i k_i$ for $i = 1, 2, \dots, N$.

This implies that the overall *F*-test examines the overall effect based on the variance of the empirical overall effect magnitudes

$[\widehat{WSEP}(x_i, k_i)]$ for $i = 1, 2, \dots, N$ indicating the weighted summed effects of predictors ($WSEP$) where the weights on the individual effects are values of the predictors. Note that the individual effects of predictors are allowed to be linked to each other in the overall effect magnitudes. In sum, the null hypothesis assessed by the overall F -test can be expressed as:

$$WSEP(x_i, k_i) = \beta x_i + \gamma k_i + \delta x_i k_i = 0 \quad \text{for } i = 1, 2, \dots, N. \quad (8)$$

From equations 3, 4, and 7, one can confirm that the overall F -test assesses the overall effect of predictors under the assumption that the true effects of predictors are exactly captured by the recovered effects of predictors.

OVERALL T -TEST

The Overall t -test at the Population Level

The recovered effect of each predictor is assumed to follow a normal distribution in regression analysis. Thus, the overall effect magnitudes (written in equation 7) can be written as:

$$WSEP(x_i, k_i) \sim N(u_{WSEP}, \sigma_{WSEP}^2) \quad \text{for } i = 1, 2, \dots, N, \quad (9)$$

$$\begin{aligned} \text{where } \hat{u}_{WSEP} &= \frac{1}{N} \sum_{i=1}^N \widehat{WSEP}(x_i, k_i) = \hat{\beta} \bar{x} + \hat{\gamma} \bar{k} + \hat{\delta} \bar{x} \bar{k}, \\ \hat{\sigma}_{WSEP}^2 &= \text{Var}(\widehat{WSEP}(x_i, k_i)) = SSR_u / (N - m - 1). \end{aligned}$$

As explained with equations 4 and 7, it is possible to diagnose the null hypothesis on the overall effect (equation 8) with the overall F -test based on the variance of the normal distribution for overall effect magnitudes. Strictly speaking, the overall F -test assesses the overall effect with a ratio of the sum of squared regression (linked to the variance of overall effect magnitudes) to the sum of squared error (linked to the variance of error terms). The two squared sums are calculated with the estimated effects of predictors. The individual effect of a predictor is not a random effect but a fixed constant in the overall F -test.

However, the mean of the overall effect magnitudes ($MOEM$) can

be specified with a normal distribution because each estimated individual effect of predictors can be considered a random effect following a normal distribution. More specifically, it is possible to represent that:

$$MOEM \sim N(u_{MOEM}, \sigma_{MOEM}^2), \quad (10)$$

where $\hat{u}_{MOEM} = E(\hat{\beta}\bar{x} + \hat{\gamma}\bar{k} + \hat{\delta}\bar{x}\bar{k}) = \beta\bar{x} + \gamma\bar{k} + \delta\bar{x}\bar{k} = \hat{u}_{WSEP}$ and:

$$\begin{aligned} \hat{\sigma}_{MOEM}^2 = Var(MOEM) &= Var(\hat{\beta})(\bar{x})^2 + Var(\hat{\gamma})(\bar{k})^2 + Var(\hat{\delta})(\bar{x}\bar{k})^2 \\ &+ 2Cov(\hat{\beta}, \hat{\gamma})(\bar{x})(\bar{k}) + 2Cov(\hat{\beta}, \hat{\delta})(\bar{x})(\bar{x}\bar{k}) + 2Cov(\hat{\gamma}, \hat{\delta})(\bar{k})(\bar{x}\bar{k}). \end{aligned}$$

Accordingly, it is possible to diagnose the null hypothesis with a test assessing whether the mean (as a random effect) is zero. We refer to this test as the overall t -test. The t -statistic for the overall t -test is written as:

$$T = \frac{E(MOEM)}{\sqrt{Var(MOEM)}} = \frac{\hat{\beta}\bar{x} + \hat{\gamma}\bar{k} + \hat{\delta}\bar{x}\bar{k}}{\sqrt{Var(\hat{\beta}\bar{x} + \hat{\gamma}\bar{k} + \hat{\delta}\bar{x}\bar{k})}} \sim t(N - m - 1), \quad (11)$$

where the variance-covariance matrix of the estimated regression coefficients is represented as $\Sigma = \hat{\sigma}^2(Z'Z)^{-1}$ where Z indicates the matrix expression of predictors (including the intercept) and $\hat{\sigma}^2 = SSE_u / (N - m - 1)$.

The t -statistic in the overall t -test is identical to that in the individual t -test when the effects of other predictors are restricted to be zero. Thus, it is possible to interpret the overall t -test as a generalized version of the individual t -test. One may conduct the overall t -test with a confidence interval of the $MOEM$. More specifically, it is possible to assess whether the mean of the overall effect is zero with a confidence interval. If the confidence interval includes zero, it is possible to conclude that the overall effect does not exist.

From equations 10 and 11, one can confirm that the overall t -test assesses the overall effect with the recovered effects as random effects, unlike the overall F -test. More specifically, the overall t -test examines the overall effect with the mean and the variance of the random effects. Furthermore, it examines the overall effect while considering the correlations among the recovered effects of

predictors (resulting from correlations among predictors).

It is notable that the multicollinearity problem indicates that it is not reliable to assess the significance of an effect separately from other effects in regression analysis. The overall t -test assesses the overall effect as a weighted sum of individual effects where the weighted sum follows a normal distribution. Thus, it is not related directly to multicollinearity due to correlated predictors. Nevertheless, it is affected by correlations among the estimated regression coefficients. If the correlations lead to a large standard error of the overall effect, the overall t -test may conclude that the overall effect is not significant even though it is diagnosed to be significant by the overall F -test.

The mean and the variance of overall effect magnitudes become zero when all individual effects of predictors are zero. Thus, one may argue that the conclusions by the overall t -test and the overall F -test should be identical. However, the overall F -test assesses the overall effect based on the recovered effects of predictors as the fixed representative measures for the true effects of predictors. Thus, it evaluates the individual effects to be zero when the recovered individual effects of predictors are zero. Furthermore, the F -statistic used in the overall F -test becomes zero when the $MOEM$ is zero. One can confirm this with equations 4, 6, 9 and 10.

In contrast, the overall t -test probabilistically assesses the overall effect based on the mean and the variances of the distribution for overall effect magnitudes. Thus, it may evaluate the individual effects to be zero even when the recovered individual effects are not zero. More specifically, the overall t -test may diagnose the overall effect to be zero (meaning that the overall effect is statistically insignificant) even if the recovered effects of predictors are not zero, because the statistical significance of the overall effect is determined by the relative magnitudes of the mean and the variance of the distribution. Accordingly, the overall t -test may be more conservative than the overall F -test. In other words, the overall t -test is expected to be more cautious in rejecting the null hypothesis than the overall F -test.

The estimated mean of overall effect magnitudes (used to calculate the variance of overall effect magnitudes in the overall F -test) is identical to the mean of the normal distribution for overall effect magnitudes (used in the overall t -test) (see equation 10). In addition, the standard error of the estimated mean (used in the overall t -test)

is a function of the variance of error term (linked to the squared sum of error used in the overall *F*-test) (see equation 11). Thus, the overall *F*-test assesses the overall effect with the variance of overall effect magnitudes adjusted by the variance of error term, whereas the overall *t*-test assesses the overall effect with the mean of overall effect magnitudes adjusted by the variance of error term.

Figure 1 helps one to intuitively understand the difference in the statistical principles between the overall *F*-test and the overall *t*-test, where the effect of a focal predictor (*X*) is assumed to be non-zero and the effects of other predictors are assumed to be zero. Then, the overall effect magnitudes of predictors are calculated with the effect magnitudes of *X*. One can measure the deviations of overall effect magnitudes from the mean and calculate the variance of overall effect magnitudes with the fixed recovered effect of *X*. The mean and the variance are used to calculate the *F*-statistic in the overall *F*-test. However, the recovered effect is assumed to be a random effect following a normal distribution in regression analysis. Accordingly, as illustrated in Figure 1, the mean of overall effect magnitudes (i.e., *MOEM* calculated at the mean of *X*) is also expressed as a random variable following a normal distribution. The mean and the variance of the normal distribution for the *MOEM* are used to calculate the *t*-statistic used in the overall *t*-test. In addition, the confidence interval for the *MOEM* can be calculated with the mean and the variance of the normal distribution for the *MOEM*. If the effect of *X* is zero, the straight line representing the overall effect magnitudes depending on values of *X* (which is a parallel line with the regression line) lies on the side of horizontal axis. Consequently, the mean of the distribution for the *MOEM* becomes zero.

In summary, the overall *F*-test assesses the overall effect based on the variance of overall effect magnitudes of predictors. The variance is zero if all individual effects are zero. The overall *t*-test assesses the overall effect based on the statistical significance of the mean of overall effect magnitudes (as a random variable). The mean is zero if all individual effects are zero. The statistics used in the overall *F*-test and the overall *t*-test are tightly linked to each other. However, they may lead to different conclusions on the overall effect. The overall *t*-test relies on the *t*-statistic calculated with the recovered effects of predictors as random effects, whereas the overall *F*-test relies on the *F*-statistic calculated with the recovered effects as fixed effects.

When there is a conflict between the individual *t*-test and the

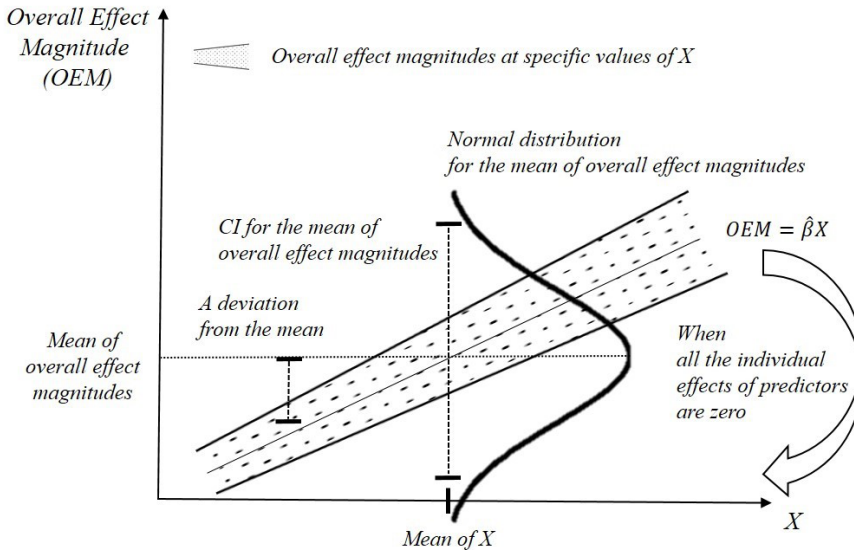


Figure 1. Graphical illustration of the overall t-test

overall F -test, one can derive the following conclusions with a further analysis by the overall t -test. First, if the overall effect is significant by the overall t -test as well as the overall F -test whereas all individual effects are not significant by the individual t -test, it is possible to interpret that the summed effect of predictors is significant even though the individual effects of predictors are not significant.

Second, the overall effect may not be significant by the overall F -test whereas an individual effect is significant by the individual t -test. If the overall effect is not significant by the overall t -test as well as the overall F -test whereas an individual effect is significant by the individual t -test, it is possible to interpret that the individual effect is not reliable due to the correlated effects of predictors.

Third, if the overall effect is significant by the overall F -test but not significant by the overall t -test, one may accept either the conclusion by the overall F -test or that by the overall t -test. One may accept the conclusion by the overall F -test if one assumes that the recovered effects are the fixed measures for the true effects. One may accept the conclusion by the overall t -test if one assumes that the recovered effects are specified with means and variance-covariances of the recovered effects. However, it is recommended to

accept the conclusion by the overall *t*-test because each recovered effect is basically assumed to follow a normal distribution.

The Overall *t*-test at Specific Values of Predictors

It is possible to assess the overall effect at specific values of predictors. The corresponding *t*-statistic is written as:

$$T_i = \frac{\widehat{WSEP}(x_i, k_i)}{\sqrt{\text{Var}(\widehat{WSEP}(x_i, k_i))}} = \frac{\hat{\beta}x_i + \hat{\gamma}k_i + \delta x_i k_i}{\sqrt{\text{Var}(\hat{\beta}x_i + \hat{\gamma}k_i + \delta x_i k_i)}} \sim t(N - m - 1) \text{ for } i=1, 2, \dots, N, (12)$$

where

$$\begin{aligned} \text{Var}(\widehat{WSEP}(x_i, k_i)) = & \text{Var}(\hat{\beta})x_i^2 + \text{Var}(\hat{\gamma})k_i^2 + \text{Var}(\delta)x_i^2k_i^2 \\ & + 2\text{Cov}(\hat{\beta}, \hat{\gamma})x_i k_i + 2\text{Cov}(\beta, \delta)x_i^2k_i + 2\text{Cov}(\gamma, \delta)x_i k_i^2. \end{aligned}$$

The subsample-specific overall *t*-test at specific values of predictors may be useful in interpreting the overall effect at the population level. The overall effect may be significant or insignificant at specific values of predictors. For example, one can confirm whether the overall effect is insignificant with the subsample-specific overall *t*-test, whereas the overall effect is diagnosed to be insignificant at the population level.

The Generalized Simple Effect

The simple slopes analysis is used for analyzing moderation effects in social sciences including management science (e.g., Finsaas and Goldstein 2020; Garden et al. 2017; Irwin and McClelland 2001; Krishna 2016; Spiller et al. 2013). To employ the simple slopes analysis, one must choose specific values of a moderator, which is referred to as a “pick-a-point” approach (Rogosa 1980). The conditional process analysis can be used to probe simple effects (e.g., Krishna 2016; Spiller et al. 2013). The simple effect is also referred to as the conditional effect in the conditional process analysis. The simple effect of a focal predictor can be interpreted as the combination of the unconditioned effect of the focal predictor and the conditioned effect of the focal predictor depending on moderator(s). It does not consider the effects of other predictors and the interaction term(s) in a regression model.

One may calculate the generalized simple effect magnitude of a focal predictor at specific values of predictors where the effects of other predictors are expressed as conditional effects of the focal predictor. Then, the overall effect magnitude of predictors is identical to the generalized simple effect magnitude of a focal predictor in the regression model.

For example, if X is the focal predictor, equation 1 can be written as:

$$y_i = \alpha + WSEP(x_i, k_i) + \varepsilon_i = \alpha + WSEP_X(x_i, k_i) x_i + \varepsilon_i \text{ for } i=1,2,\dots,N, \quad (13)$$

where

$$WSEP(x_i, k_i) = \beta x_i + \gamma k_i + \delta x_i k_i \text{ and } WSEP_X(x_i, k_i) = \beta + \gamma(k_i / x_i) + \delta k_i.$$

One can confirm that the overall effect magnitude of predictors [i.e., $WSEP(x_i, k_i)$] is identical to the generalized simple effect magnitude of the focal predictor X [i.e., $WSEP_X(x_i, k_i)x_i$] at the specific values of the two predictors (i.e., x_i and k_i). Furthermore, one can interpret $WSEP_X(x_i, k_i)$ as the generalized simple effect of X at specific values of the two predictors where β , $\gamma(k_i/x_i)$, and δk_i capture the main effect of X , the conditional effect of X capturing the main effect of K , and the conditional effect of X through the interaction between X and K .

Subsequently, one can confirm that the test for the specific overall effect magnitude of predictors is identical to the test for the generalized simple effect of the focal predictor X in the regression model. For example, if X is the focal predictor,

$$\begin{aligned} T_i &= \frac{\hat{\beta}x_i + \hat{\gamma}k_i + \delta x_i k_i}{\sqrt{\text{Var}(\hat{\beta}x_i + \hat{\gamma}k_i + \delta x_i k_i)}} = \frac{x_i(\hat{\beta} + \hat{\gamma}(k_i/x_i) + \delta k_i)}{x_i \sqrt{\text{Var}(\hat{\beta} + \hat{\gamma}(k_i/x_i) + \delta k_i)}} \\ &= \frac{\hat{\beta} + \hat{\gamma}(k_i/x_i) + \delta k_i}{\sqrt{\text{Var}(\hat{\beta} + \hat{\gamma}(k_i/x_i) + \delta k_i)}} \text{ for } i = 1, 2, \dots, N. \end{aligned} \quad (14)$$

This implication seems reasonable because it is assumed that the effect magnitude of a predictor can be substituted by the effect magnitudes of other predictors in a compensatory model.

The Scale Effect

The mean of overall effect magnitudes used in the overall t -test is not scale-invariant. Thus, the overall t -test is affected by

transformation of observations. For example, the mean of overall effect magnitudes is transformed to be zero in mean-centered or standardized regression models. In addition, the overall t -test allows the summed effect of some predictors to be balanced out by the summed effect of other predictors.

Thus, one may assess the overall effect with a transformed regression model in which all effect directions of predictors are identical. Furthermore, in the transformed regression model, the mean of the overall effect magnitudes (used in the overall t -test) is more consistent with the overall effect in the null hypothesis written as equation 2. The null hypothesis describes the overall effect with the individual effects that are not balanced out due to the different effect directions of predictors. The F -statistic used in the overall F -test is not changed in the transformed regression model. Accordingly, it is possible to compare the conclusions by the overall t -test and the overall F -test in the transformed regression model.

In sum, when one investigates the conflict between the individual t -test and the overall F -test, one can compare the overall t -test and the overall F -test with a regression model in which the overall effect may be partially balanced out due to different effect directions of predictors. One can also use a transformed regression model in which all effect directions are identical. However, it is recommended to use the transformed regression model if one wants to assess the overall effect consistently with the null hypothesis written as equation 2.

A SIMULATED ILLUSTRATION

This paper does not insist that the overall t -test is superior to the overall F -test. Rather, this paper recommends one to use the overall t -test to further investigate a conflict between the individual t -test and the overall F -test. Accordingly, a simulation was designed to illustrate that the conclusion by the overall t -test may be identical to or different from that by the overall F -test.

Simulation Procedure

We generated data sets with a simulation model written as:

$$y_i = \alpha + \beta x_i + \gamma k_i + \delta k_i x_i + \varepsilon_i \text{ for } i = 1, 2, \dots, N, \quad (15)$$

where $\begin{pmatrix} x_i \\ k_i \end{pmatrix} \sim BN \left(\begin{pmatrix} \mu_x \\ \mu_k \end{pmatrix}, \begin{pmatrix} \tau_x^2 & \tau_{x,k} \\ \tau_{x,k} & \tau_k^2 \end{pmatrix} \right)$ and $\varepsilon_i \sim N(0, \sigma^2)$,

further where $N = 200$, $\begin{pmatrix} \mu_x \\ \mu_k \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$, and $\begin{pmatrix} \tau_x^2 & \tau_{x,k} \\ \tau_{x,k} & \tau_k^2 \end{pmatrix} = \begin{pmatrix} 3 & .1 \\ .1 & 2 \end{pmatrix}$,

$\alpha = 1.0$, $\beta = .08$, $\gamma = .06$, $\delta = .0025$, and $\sigma^2 = .1$ or $.5$.

In this simulation model, we set the effects of predictors to be marginal (i.e., $\beta = .08$, $\gamma = .06$, and $\delta = .0025$). One may argue that there exists an overall effect because the individual effects of predictors are non-zero. It may be true in view of the statistical principle used in the overall F -test, because the overall F -test assumes that the recovered effects (i.e., estimated regression coefficients) of predictors represent the true effects of predictors. However, it may not be true in view of the statistical principle used in the overall t -test and the individual t -test. The two t -tests do not assume that the recovered effect of a predictor exactly captures the true effect. They consider the recovered effect as a random effect that can be specified with a normal distribution. The overall t -test assumes that the recovered effect of a predictor may be correlated with the recovered effects of other predictors, whereas the individual t -test assumes that the recovered effect of a predictor is independent from the recovered effects of other predictors.

If predictors are correlated, conclusions on the effects of predictors might differ between the overall F -test and the individual t -test. We set the correlation between the two predictors to be very low ($\text{Corr}(x, k) = .1 / (\sqrt{3}\sqrt{2}) = .041$). Note that the predictors may have considerable multicollinearity in the simulation model although the correlation is very low, because the regression model includes the interaction term of the two predictors.

The variance-covariances of regression coefficients in the simulation model are affected by the randomness in simulated observations (i.e., the variance-covariances of predictors) and the level of simulated error variance. The variance-covariances are affected critically by error variance (i.e., σ^2), as mentioned above. To confirm the effect of error variance, we generated two data types according to two levels of σ^2 ($\sigma^2 = .1$ vs $.5$). One can expect that the

recovered effects are more likely to be significant in the low variance condition (i.e., $\sigma^2 = .1$) than in the high variance condition ($\sigma^2 = .5$).

The simulated data sets include 200 observations. We use the sample size of 200 observations, which has been used in previous behavioral research (Aguinis, Edwards, and Bradley 2017). This simulation was conducted with R as a programming language.

Estimation and Testing

We examined the individual effects with the individual *t*-test, and the overall effect with the overall *F*-test and the overall *t*-test. In addition, we have examined the overall effects of predictors with the subsample-specific overall *t*-test at various values of predictors. More specifically, we have examined the overall effects at 10th, 25th, 50th, 75th, and 90th percentiles of the two predictors.

It is easy to obtain the *t*-statistics in the individual *t*-test and the *F*-statistics in the overall *F*-test, because they are provided by any statistical software. However, the *t*-statistics used in the overall *t*-test should be calculated with the formula in equations 11 and 12. This implies that it is necessary to get the estimation results of the regression model. The results were obtained from the OLS estimation.

Simulation Results

The results showed that the overall *t*-test evaluates the overall effect more conservatively than the overall *F*-test, as we have explained with the statistical principles used in the two tests. We summarized two cases of the simulation results in Tables 1 and 2. We interpreted the results where the significance level is .001.

Table 1 (presenting results in the low variance condition) shows that there was a multicollinearity problem. The *VIFs* (Variance Inflation Factors) for a predictor and the interaction term were higher than 10 (which is the criterion evaluating the degree of multicollinearity). There is no problem to assess the overall effect with the overall *F*-test and the overall *t*-test, because the two tests do not suffer from multicollinearity. The overall effect was significant with the overall *t*-test as well as the overall *F*-test. In addition, the subsample-specific overall *t*-test showed that the overall effect was significant at all the analyzed percentiles of the two predictors.

Table 1. Simulation results in the low variance condition ($\sigma^2 = .1$)

Analysis I	α	β	γ	δ	F	R^2
Estimate	.598	.156	.101	.017	283.7***	.813
t -statistic	1.239	1.925	2.142*	2.157*		
VIF	-	42.810	8.993	49.993		
Analysis II	X	K	Value of X	Value of K	WSEP	Ti
Subsample Level	10th-PT	10th-PT	3.093	8.219	1.746	3.469***
	10th-PT	25th-PT	3.093	8.981	1.863	3.458***
	10th-PT	50th-PT	3.093	9.810	1.990	3.443***
	10th-PT	75th-PT	3.093	10.913	2.159	3.421***
	10th-PT	90th-PT	3.093	11.708	2.282	3.405***
	25th-PT	10th-PT	4.049	8.219	2.029	3.548***
	25th-PT	25th-PT	4.049	8.981	2.159	3.550***
	25th-PT	50th-PT	4.049	9.810	2.300	3.547***
	25th-PT	75th-PT	4.049	10.913	2.487	3.537***
	25th-PT	90th-PT	4.049	11.708	2.622	3.527***
	50th-PT	10th-PT	5.411	8.219	2.433	3.562***
	50th-PT	25th-PT	5.411	8.981	2.580	3.579***
	50th-PT	50th-PT	5.411	9.810	2.740	3.590***
	50th-PT	75th-PT	5.411	10.913	2.953	3.594***
	50th-PT	90th-PT	5.411	11.708	3.107	3.592***
	75th-PT	10th-PT	6.345	8.219	2.710	3.542***
	75th-PT	25th-PT	6.345	8.981	2.869	3.566***
	75th-PT	50th-PT	6.345	9.810	3.042	3.583***
	75th-PT	75th-PT	6.345	10.913	3.273	3.594***
	75th-PT	90th-PT	6.345	11.708	3.439	3.597***
	90th-PT	10th-PT	7.512	8.219	3.056	3.505***
	90th-PT	25th-PT	7.512	8.981	3.230	3.534***
	90th-PT	50th-PT	7.512	9.810	3.420	3.556***
	90th-PT	75th-PT	7.512	10.913	3.673	3.573***
	90th-PT	90th-PT	7.512	11.708	3.855	3.580***
Population Level	N=200, Mean(X)=5.279, Mean(K)=9.930, Mean(XK)=52.367, MOEM=2.217, T=3.590***					

Note 1: *** $p < .001$, ** $p < .01$, * $p < .05$.

Note 2: PT indicates the percentile of the two simulated predictors (X and K).

Note 3: F indicates the F -statistic used in the overall F -test.

Note 4: T indicates the t -statistic used in the overall t -test.

Note 5: Ti indicates the t -statistic used in the subsample-specific overall t -test.

Note 6: VIF was calculated for the predictor corresponding to the regression coefficient.

Table 2. Simulation results in the high variance condition ($\sigma^2 = .5$)

Analysis I	α	β	γ	δ	F	R^2
Estimate	.945	.162	.054	.019	59.4***	.468
<i>t</i> -statistic	.841	.787	.479	.936		
<i>VIF</i>	-	52.531	9.877	63.172		
Analysis II	X	K	Value of X	Value of K	<i>WSEP</i>	Ti
Subsample Level	10th-PT	10th-PT	2.839	8.268	1.358	1.130
	10th-PT	25th-PT	2.839	8.977	1.435	1.121
	10th-PT	50th-PT	2.839	9.998	1.546	1.106
	10th-PT	75th-PT	2.839	10.992	1.655	1.093
	10th-PT	90th-PT	2.839	11.975	1.761	1.081
	25th-PT	10th-PT	3.728	8.268	1.644	1.206
	25th-PT	25th-PT	3.728	8.977	1.733	1.200
	25th-PT	50th-PT	3.728	9.998	1.862	1.190
	25th-PT	75th-PT	3.728	10.992	1.987	1.180
	25th-PT	90th-PT	3.728	11.975	2.110	1.170
	50th-PT	10th-PT	5.114	8.268	2.089	1.264
	50th-PT	25th-PT	5.114	8.977	2.197	1.264
	50th-PT	50th-PT	5.114	9.998	2.353	1.261
	50th-PT	75th-PT	5.114	10.992	2.505	1.255
	50th-PT	90th-PT	5.114	11.975	2.655	1.249
	75th-PT	10th-PT	6.388	8.268	2.498	1.286
	75th-PT	25th-PT	6.388	8.977	2.623	1.289
	75th-PT	50th-PT	6.388	9.998	2.804	1.290
	75th-PT	75th-PT	6.388	10.992	2.980	1.289
	75th-PT	90th-PT	6.388	11.975	3.154	1.285
	90th-PT	10th-PT	7.205	8.268	2.760	1.292
	90th-PT	25th-PT	7.205	8.977	2.897	1.297
	90th-PT	50th-PT	7.205	9.998	3.094	1.300
	90th-PT	75th-PT	7.205	10.992	3.285	1.300
	90th-PT	90th-PT	7.205	11.975	3.475	1.298
Population Level	N=200, Mean(X)=5.084, Mean(K)=10.046, Mean(XK)=51.177, MOEM=2.351, T=1.260					

Note 1: *** $p < .001$, ** $p < .01$, * $p < .05$.

Note 2: *PT* indicates the percentile of the two simulated predictors (X and K).

Note 3: F indicates the F -statistic used in the overall F -test.

Note 4: T indicates the t -statistic used in the overall t -test.

Note 5: Ti indicates the t -statistic used in the subsample-specific overall t -test.

Note 6: *VIF* was calculated for the predictor corresponding to the regression coefficient.

However, the effects of the two predictors and the interaction term were not significant with the individual *t*-test.

Table 2 (presenting results in the high variance condition) shows that there was a multicollinearity problem. The *VIFs* for one predictor and the interaction term were higher than 10. The effects of the two predictors and the interaction term were not significant with the individual *t*-test. The overall effect was diagnosed differently between the two overall tests. It was significant with the overall *F*-test, but insignificant with the overall *t*-test. The subsample-specific overall *t*-test showed that the overall effect was insignificant at all the analyzed percentiles of the two predictors.

Table 2 illustrates a case that the subsample-specific overall *t*-test diagnoses the overall effect to be insignificant at all the analyzed percentiles of the two predictors, although the overall *F*-test diagnoses the overall effect to be significant. The overall effect at specific values of the predictors indicates the weighted summed effect magnitude of the predictors at the specific values of the predictors. Accordingly, the subsample-specific overall *t*-test shows that the overall effect was insignificant at all the analyzed percentiles of the two predictors. In addition, the overall effect was insignificant at all possible values of the two predictors. This is contradictory to the implication by the overall *F*-test. The overall effect cannot be insignificant at all possible values of the two predictors if the overall effect is truly significant. Thus, the case shows that it may not be sufficient to examine the overall contribution of predictors only with the overall *F*-test.

CONCLUSIONS

Previous research has examined the effects of predictors with the individual *t*-test and the overall *F*-test. However, the overall *F*-test may not reject the overall effect even when the individual *t*-test rejects all individual effects of predictors. It is common to interpret that the conflict between the overall *F*-test and the individual *t*-test results from the correlations among predictors.

This paper shows that the conflict may result from different views on the recovered effects of predictors. The overall *F*-test assumes that the recovered effects exactly capture the true effects of predictors. In contrast, the individual *t*-test evaluates effects of

predictors as random effects. This paper presents the overall t -test assessing the overall effect and recommends one to use the overall t -test to investigate a conflict between the individual t -test and the overall F -test. More specifically, it is possible to assess the overall effect based on the mean and the variance of the normal distribution representing the overall effect magnitudes. The overall F -test assesses the overall effect based on the variance of the distribution, whereas the overall t -test assesses the overall effect based on the mean of the distribution.

In sum, if there exists a conflict between the individual t -test and the overall F -test, one may conduct the overall t -test. It is recommended to accept the conclusion by the overall t -test when the conclusion by the overall t -test differs from that by the overall F -test.

The overall t -test can help researchers to understand a conflict between the overall F -test and the individual t -test. It is consistent with the individual t -test in that it evaluates the effects of predictors with the distributions of effects. It is also consistent with the overall F -test in that it evaluates the overall effect as a whole. This paper has explained the overall t -test and the overall F -test with a multiple regression model estimated with ordinary least squares. This implies that the regression model requires the assumption of homoscedasticity. If this assumption is violated, one may use the two overall tests using a regression model estimated with weighted least squares or generalized least squares.

Regression analysis summarizes the results with two tables. One is the ANOVA table reporting the estimation results under fixed effects of predictors. The ANOVA table includes the F -statistic for the overall effect in the overall F -test. The other is the regression table reporting the estimation results under random effects of predictors. The regression table shows the estimated effect of each predictor with the corresponding t -statistic used in the individual t -test.

ANOVA statistically diagnoses whether a dependent variable is different across groups (such as gender and nation). It is common to assess the difference between two groups with a t -test. However, it is not easy to assess the differences among three or more groups with a t -test. Thus, previous research has used ANOVA to assess the differences among three or more groups. ANOVA provides only an overall summary of estimation results with the ANOVA table. It does not report the statistics for estimated differences (corresponding to

the specific effects of predictors), because the estimated differences are assumed to be fixed effects. The results from post-hoc analysis (as a subsequent analysis of ANOVA) correspond to those in the regression table, because the post-hoc analysis assesses the differences as random effects with various versions of *t*-test.

In the regression model corresponding to ANOVA, the intercept captures the mean of a reference group. The effect of each dummy variable captures the mean difference between the reference group and the compared group. Thus, the overall *F*-test in ANOVA assesses the omnibus effect of dummy variables (capturing the overall difference among groups). In addition, a *t*-test used in the post-hoc analysis assesses the differences between the intercept and the summed effect of the intercept and each dummy variable.

Accordingly, the implications presented in this paper can be applied to ANOVA. More specifically, there may exist a conflict between ANOVA and post-hoc analysis. In other words, the conclusions on the differences for a pair of groups (by the post-hoc analysis) may not be consistent with the conclusions on the differences among all groups (by ANOVA). When there exists a conflict between ANOVA and the post-hoc analysis, one may conduct the overall *t*-test.

One can represent levels of a categorical predictor with dummy variables and recover the effects of dummy variables with a regression model. Then, one can assess whether a dependent variable is different across groups with the overall *t*-test. If the effect directions of the dummy variables are not identical, one can use a transformed regression model in which all effect directions are identical. In the transformed regression model, the mean of the normal distribution is expressed as a sum of the recovered effects (re-scaled to be non-negative). Thus, if the sum is not significant by the overall *t*-test, one can conclude that the dependent variable is not different across all groups. We leave this issue on the conflict between ANOVA and the post-hoc analysis for future research.

If a regression model suffers from a multicollinearity problem, the individual *t*-test may lead to a false judgment on individual effects. It is a traditional approach to look at the *F*-test result or *R*² to see whether the overall effect is reliable. If the overall effect appears strong, one may derive a parsimonious model by dropping some redundant variables based on theories and/or intuition/previous research findings.

However, as explained before, researchers may want to examine the effects of all predictors without dropping some redundant variables like in a moderated regression model which may suffer from multicollinearity. To alleviate the multicollinearity problem, they may use the simple slopes analysis. It is notable that the *t*-test using simple slopes used in the simple slopes analysis is a special case of the overall *t*-test, as explained in the section for generalized simple effect.

The overall *t*-test proposed in this paper is not more convincing than the *F*-test used in the traditional approach. The overall *t*-test is a supplementary test of the *F*-test. The overall *t*-test can assess the overall effect like the *F*-test regardless of multicollinearity. However, the overall *t*-test and the overall *F*-test evaluate the overall effect with different views on the overall effect. The overall *F*-test assesses the overall effect of predictors in view of the variance of overall effect. In contrast, the overall *t*-test assesses the overall effect in view of the mean of overall effect.

If previous research reports conflicting implications between the individual *t*-test and the *F*-test, one may reexamine the conflicting implications with the overall *t*-test. It is possible to re-interpret the conflicting implications with the overall *t*-test. It would be valuable to empirically show that the conflicting implications between the overall *F*-test and the individual *t*-test on statistical conclusions or inferences can be resolved with the overall *t*-test.

REFERENCES

- Aguinis, H. and J. C. Beaty (2005), "Effect Size and Power in Assessing Moderating Effects of Categorical Variables Using Multiple Regression: A 30-Year Review," *Journal of Applied Psychology*, 90(1), 94-107.
- _____, J. R. Edwards, and K. J. Bradley (2017), "Improving Our Understanding of Moderation and Mediation in Strategic Management Research," *Organizational Research Methods*, 20(4), 665-685.
- Aiken, L. S. and S. G. West (1991), *Multiple Regression: Testing and Interpreting Interactions*, Newbury Park, CA: Sage Publications.
- Cronbach, L. J. (1987), "Statistical Tests for Moderator Variables: Flaws in Analyses Recently Proposed," *Psychological Bulletin*, 102(3), 414-417.
- Echambadi, R. and J. D. Hess (2007), "Mean-centering Does Not Alleviate Collinearity Problems in Moderated Multiple Regression Models,"

- Marketing Science*, 26(3), 438-445.
- Ernst, A. F. and C. J. Albers (2017), "Regression Assumptions in Clinical Psychology Research Practice - A Systematic Review of Common Misconceptions," *PeerJ*, 5, e3323.
- Finsaas, M. C. and B. L. Goldstein (2020), "Do Simple Slopes Follow-up Tests Lead Us Astray? Advancements in the Visualization and Reporting of Interactions," *Psychological Methods*, 26(1), 38-60.
- Hayes, A. F. (2013), *Introduction to Mediation, Moderation, and Conditional Process Analysis: A Regression Based Approach*. New York: The Guilford Press.
- Irwin, J. R. and G. H. McClelland (2001), "Misleading Heuristics and Moderated Multiple Regression Models," *Journal of Marketing Research*, 38(1), 100-109.
- Krishna, A. (2016), "A Clearer Spotlight on Spotlight: Understanding, Conducting and Reporting," *Journal of Consumer Psychology*, 26(3), 314-324.
- Lamberts, K. (2000), "Information-accumulation Theory of Speeded Categorization," *Psychological Review*, 107(2), 227-260.
- Park, S.-J. and Y. Yi (2023), "Decomposing Main Effects in Moderated Regression Models," *Journal of Business Research*, 157, <https://doi.org/10.1016/j.jbusres.2022.113577>.
- Rogosa, D. (1980), "Comparing Nonparallel Regression Lines," *Psychological Bulletin*, 88(2), 307-321.
- Rubin, D. C., S. Hinton, and A. Wenzel (1999), "The Precise Time Course of Retention," *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 25(5), 1161-1176.
- Sharma, S., R. M. Durand, and O. Gur-Arie (1981), "Identification and Analysis of Moderator Variables," *Journal of Marketing Research*, 18(3), 291-300.
- Spiller, S. A., G. J. Fitzsimons, J. G. Lynch Jr., and G. H. McClelland (2013), "Spotlights, Floodlights, and the Magic Number Zero: Simple Effects Tests in Moderated Regression," *Journal of Marketing Research*, 50(2), 277-288.
- Usher, M. and J. L. McClelland (2001), "The Time Course of Perceptual Choice: The Leaky, Competing Accumulator Model," *Psychological Review*, 108(3), 550-592.
- Wilcox, R. R. (2008), "Post-hoc Analyses in Multiple Regression Based on Prediction Error," *Journal of Applied Statistics*, 35(1), 9-17.

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