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Financial Equilibrium with Heterogeneous Information Processing Efficiency

JUNGSUK HAN*
Seoul National University
Seoul, Korea

ABSTRACT

This paper examines a model where investors' varying information processing abilities influence financial market equilibrium through price informativeness. When prices are sufficiently informative, high-efficiency investors specialize in high-signal-efficiency assets, while low-efficiency investors rely on price information and specialize in low-signal-efficiency assets. Consequently, assets with low signal efficiency exhibit higher risk premiums compared to those with high signal efficiency. This suggests that individuals with lower information processing efficiency may hold more small stocks with less efficient signals, potentially leading to higher risk premiums in small stocks compared to larger ones, driven by information-related factors.

1. INTRODUCTION

Numerous studies in financial economics emphasize that information asymmetry is a key factor shaping equilibrium in financial markets. As noted by Grossman and Stiglitz (1980),

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Correspondence Information: Jungsuk Han, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul, 08826, Korea, E-mail: jungsuk.han@snu.ac.kr

trading arises due to variations in risk aversion, endowments, or beliefs (information). However, limited research has explored the origins of information asymmetry in financial markets. It may stem from initial information endowments, but this explanation alone is insufficient as agents can actively gather information. Other factors likely contribute to information asymmetry. Notably, differences in agents' information processing capacities appear to be a significant driver of information asymmetry in financial markets.

If different capacities influence information asymmetries, which in turn determine investors' trading behavior, we can anticipate that investors with similar information processing capacities will exhibit comparable trading behavior, controlling for other key factors such as wealth endowment and risk preferences. Empirical evidence supports this notion, as Barber and Odean (2000) observed that households tend to favor small, high-beta stocks in their investment choices. Additionally, Gompers and Metrick (2001) found that large institutions, in contrast to other investors, prefer investments in large stocks with low historical returns. Furthermore, Lee and Kumar (2006) argue that retail investors' trading behavior is correlated. Despite these observations, few studies have provided explanations for why individual investors tend to exhibit similar trading patterns and investment holdings compared to institutional investors.

In this paper, I examine a model in which investors possess varying information processing capacities. The investor pool comprises two distinct groups: one with higher efficiency, representing institutions, and the other with lower efficiency, representing individual investors. While empirical evidence may warrant further investigation, this conjecture appears reasonable, given the widely accepted understanding that professional investors typically possess specialized skills and dedicate more time and effort to information collection and processing.

I develop a simple framework for information processing in this paper, which can feature heterogeneous efficiencies of information processing. The model is flexible enough to accommodate different efficiencies across various assets and individuals. This framework differs from the standard information theory approach that quantifies information processing efficiency using entropy. For example, in the seminal paper, Sims (2003) connects information theory to agents' utility maximization problems. Peng (2005) also demonstrates a financial equilibrium model incorporating information efficiency constraints. Using the framework, I investigate how the diversity in investors' information efficiency influences financial equilibrium, particularly when the informational complexities of signals vary across different assets.

I employ a noisy Rational Expectation Equilibrium (REE) model, similar to the one described in Grossman and Stiglitz (1980) and Hellwig (1980). My model diverges from the conventional setup by integrating two separate investor groups with varying capacities for information acquisition, alongside two distinct assets characterized by differing costs associated with information processing. The group of investors with high processing capacity is interpreted as sophisticated investors, such as professionals or financial institutions, while those with low processing capacity are considered individual (or retail) investors. The asset characterized by high information processing efficiency is regarded as a major stock, abundant in available information, while the one with low efficiency is seen as a minor stock, possessing minimal available information.

In my findings using this framework, I find that investors with lower information efficiency (i.e., individual investors) allocate a greater proportion of their information efficiency to assets with lower signal efficiency in equilibrium, as opposed to assets with higher signal efficiency. Consequently, low-efficiency investors exhibit a bias towards investing in assets with lower signal efficiency. Conversely, high-efficiency investors tend to favor assets with higher signal efficiency. Given the observed positive correlation between firm size and signal efficiency, this phenomenon can help explain why individual investors tend to hold more small stocks.

Certainly, liquidity or transaction costs could potentially explain this phenomenon. However, in this paper, I demonstrate that the diversity in information processing efficiency significantly influences investors' portfolio choices, even in the absence of liquidity or transaction costs.

As a corollary of the main result, I illustrate that several intriguing applications concerning individuals' portfolios can be derived from this finding. Firstly, the size effect observed in the market could be elucidated by the heterogeneity of information processing efficiency. Secondly, the initial endowment of information can influence capacity allocation, leading investors to allocate more capacity to assets with the initial information endowment. This outcome could provide insights into limited agent participation in the stock market and the prevalence of home bias in international financial markets.

The results also shed light on why fundamental analysis continues to yield positive results. Despite market efficiency principles suggesting that all public information should be reflected in prices, numerous studies argue that fundamental analysis can lead to abnormal returns. This discrepancy can be attributed to investors' limited information processing capacity, preventing the utilization of all available information. Some information is inevitably overlooked due to these limitations. Additionally, this paper predicts that information related to small firms is more likely to be overlooked compared to that of larger firms.

Lastly, I discover that the initial endowment of information influences equilibrium choices, effectively locking agents into choices where they specialize in assets with their initial information endowment. This result offers insight into explaining the phenomenon of home bias in international financial markets.

2. THE MODEL

2.1. The economy

Consider an economy spanning two periods, denoted as $t = 1, 2$. This model comprises investors categorized into two groups based on their information processing efficiencies. For simplicity, I will refer to investors with high efficiency as "high type" (H) and those with lower efficiency as "low type" (L). High-type investors can process market-generated information more efficiently, enabling them to make more precise predictions regarding the returns of risky assets with considerably less effort than low-type investors. We assume that there is a unit mass of high types, denoted as μ, and a unit mass of low types, represented as $1 - \mu$, in the economy.

Within this economy, there are three categories of assets: a riskless asset and two risky assets. One of the risky assets possesses greater signal efficiency, while the other exhibits lower signal efficiency. Consequently, the asset with higher informational efficiency will yield more valuable information for the same level of investment effort. I will refer to the former as the "efficient asset" (E) and the latter as the "inefficient asset" (I). It's worth noting that these assets can also be conceptualized as portfolios of efficient or inefficient assets, rather than individual assets. For each investor, I will represent their holdings of each asset type as X_F (riskless asset), X_E (efficient asset), and X_I (inefficient asset).

I set the price of the riskless asset equal to unity and denote the prices of the risky assets as P_E and P_I for the efficient asset and the inefficient asset, respectively. The riskless asset yields a return of R per unit at the end of period two, while the risky assets generate uncertain returns, denoted as u_E and u_I for the efficient and the inefficient assets, respectively. The return for the efficient asset, u_E, consists of two components:

$$u_E = \theta_E + \epsilon_E,$$

where θ_E is observable given the correct signals, and ϵ_E is unobservable. Similarly, the return for the inefficient asset, u_I, also comprises two components:

$$u_I = \theta_I + \epsilon_I,$$

where θ_I is observable given the correct signals, and ϵ_I is unobservable. Finally, I assume that θ_E, θ_I, ϵ_E, and ϵ_I are all independent of each other. Additionally, I assume that θ_E and θ_I follow an i.i.d. normal distribution with a mean of $\overline{\theta}$ and a variance of σ_θ^2, while ϵ_E and ϵ_I follows an i.i.d. normal distribution with a mean of zero and a variance of σ_ϵ^2.

Each investor is endowed with a fixed amount of wealth W_1, and makes a decision on his capacity allocation as well as an investment decision at $t = 1$. At time $t = 2$, he observes the realized return of each asset. An investor with portfolio (X_F, X_E, X_I) will have

$$W_2 = RX_F + u_E X_E + u_I X_I.$$

Every investor in the economy has the same utility function $V(W)$. I assume a constant absolute risk aversion (CARA) utility function,

$$V(W) = -e^{-aW}, \quad a > 0,$$

where a is the coefficient of absolute risk aversion. I normalize the initial wealth of investors to zero without loss of generality.

I assume that the per capita supply of risky assets are x_E and x_I for the efficient and the inefficient assets, respectively. I also assume that x_E and x_I follow an i.i.d. normal distribution with mean $\bar{x}$ and variance σ_x^2 where $\bar{x} > 0$. Finally, the market clearing conditions are given by

$$\mu X_E^H + (1 - \mu) X_E^L = x_E;$$

and

$$\mu X_I^H + (1 - \mu) X_I^L = x_I.$$

2.2. Heterogeneous Information Processing Abilities

I incorporate the concept of information theory into the investor's utility maximization problem to account for information processing capacity. The approach here can be motivated by information processing frameworks developed by Sims (2003) and Peng (2005), who study agents' information processing with different channel capacities. In this context of information theory, the quantity of information conveyed by a signal is quantified by the reduction in uncertainty achieved by that signal.

In this paper, I do not assume that the transmission of the signal achieves this optimal level. Because human language lacks the flexibility of communication theory, achieving optimal encoding is not feasible. For instance, consider two firms with independent but identical stock price distributions. According to Shannon's theorem in information theory, the transmission of signals about the price information of these two firms should incur the same amount of entropy. This result is based on the assumption of an optimal coding technique for signal transmission. However, one company might have a more efficient signal regarding its price information, while the other might have a less efficient one. The reason we cannot reach the Shannon limit of information transmission is that human communication lacks the required flexibility for optimal coding.

Therefore, in this financial information transmission setting, I deviate from the optimal coding framework and instead assume that the amount of information transmitted can be determined based on the signal's efficiency, regardless of the signal's entropy. In other words, the amount of entropy in a signal is no longer equivalent to the amount of uncertainty reduced by the signal. Building on the above argument, I establish the following definition.

An asset is considered to have higher *signal efficiency* when the ratio of uncertainty reduced to the entropy of the signal, also known as the *information-cost ratio*, is higher. This information-cost ratio is defined as:

$$\text{Information-cost ratio} = \frac{\text{Amount of uncertainty reduced}}{\text{Amount of processing resource consumed}}$$

Below, I develop a model for allocating information processing resources according to the previously defined criteria. I assume that each investor can utilize their financial wealth (including the opportunity cost of their human capital) for information processing. Additionally, I assume they are limited by the maximum processing capacity of a single signal unit. In essence, they can select only one signal for either the efficient or inefficient asset, each potentially incurring different costs.

Since they differ in their processing efficiencies, the financial costs vary across the types. To obtain a single signal on asset $k \in \{E, I\}$,, the financial cost for each type, denoted as $\tau \in L, H$, is represented by $C_k^{\tau} > 0$, where

$$C_k^H \leq C_k^L \text{ for all } k \in \{E, I\};$$

and

$$C_E^{\tau} \leq C_I^{\tau} \text{ for all } \tau \in \{H, L\}.$$

In other words, the cost of acquiring information processing resources can be higher for low type investors than for high type investors, and is also significantly higher for the inefficient asset than for the efficient asset. For simplicity, I assume that the inefficient asset costs the same for both types of investors ($C_I^H = C_I^L = C_I$). Therefore, the wedge in the costs of processing information between two assets, denoted as $\Delta C^{\tau} = C_I - C_E^{\tau}$, represents the difference in information processing efficiencies between the two groups. Note that the disparity is considerably larger for low types than for high types because the efficiency gain from collecting information on the efficient asset is greater for high types:

$$\Delta C^H > \Delta C^L > 0.$$

It can be interpreted that signals are more challenging to interpret for the inefficient asset than for the efficient asset, given the same amount of resource allocation for information processing. Furthermore, this difficulty matters more for low-type investors than for high-type ones.

According to the initial assumptions on signals, signals are not differentiated among different investors. Therefore, if an investor receives a larger number of signals than another investor, they have the same information as the other investor and additional information that the other investor does not have.

I introduce a few concepts that will be frequently used in this paper. Here, a "group" of investors refers to any set that has a positive measure, meaning it is not an individual, which has a measure of zero. For example, the group of high-type investors, denoted as H, and the group of low-type investors, denoted as L, also belong to the set I. Building on the concepts defined above, I define informational dominance as follows,

An investor group i is informationally dominating an investor group j for a risky asset k if group i's information resource allocation on the specific asset k, is larger than that of group j's. On the other hand, the group j is informationally dominated by group i for a risky asset k.

If a group is informationally dominating all other investors, they do not acquire any new information from the price. In other words, they solely rely on their own signals for inferring the mean and variance of the return. On the contrary, informationally dominated investors utilize both the price and their own signals for learning purposes.

2.3. Equilibrium in Financial Market given Capacity Allocations

In this subsection, I investigate investors' optimal portfolio choice and the resulting equilibrium prices using standard REE framework.

As in the previous section, I denote investor i's private information on the efficient asset as s_E and private information on the inefficient asset as s_I. I additionally use Ø to represent the absence of private information on a specific asset. A type-τ investor's expected utility given his private signal $s^i \in \{s_E, s_I\}$ and the vector of prices $P = [P_E\ P_I]$ is given by

$$
\begin{aligned}
E[V(W\ )\mid s\ , P] &= -\exp\left(-a\{E[W_2 \mid s^i, P] - - Var[W_2 \mid s^i, P]\right) \\
&= -\exp\left(-a[-RC_k + X_E(E[u_E \mid s_E, P_E] - RP_E\right) \\
&\quad + X_I(E[u_I \mid s_I^i, P_I] - RP_I) - - (X_E Var[u_E \mid s_E^i, P_E] \\
&\quad\ X_I Var[u_I \mid s_I, P_I]))]).
\end{aligned}
$$

Maximizing the expected utility with respect to X_E and X_I yields the following demand functions for risky assets:

$$
X_E(s_E^i, P_E] = \frac{E[u_E \mid s_E^i, P_E] - RP_E}{aVar[u_E \mid s_E^i, P_E]};
$$

$$
X_I(s_I^i, P_I] = \frac{E[u_I \mid s_I^i, P_I] - RP_I}{aVar[u_I \mid s_I^i, P_I]},
$$

where $s_E^i \in \{s_E, \emptyset\}$ and $s_I^i \in \{s_I, \emptyset\}$ depending on the choice of information processing.

The equilibrium prices can be obtained by solving the following market clearing conditions:

$$
\upsilon_E X_E(s_E, P_E) + (1 - \upsilon_E) X_E(\emptyset, P_E) = x_E;
$$

and

$$
\upsilon_I X_E(s_I, P_I) + (1 - \upsilon_I) X_E(\emptyset, P_I) = x_I,
$$

where υ_k denotes the mass of investors who collect a signal on asset $k \in \{E, I\}$.

We denote by υ_k the mass of informationally dominant group on the risky asset k. By employing the optimal portfolios and the market clearing conditions, equilibrium prices can be derived as follows:

Theorem 1. *Given any allocation of information processing resource, there exists an equilibrium in the financial market for all* $k \in \{E, I\}$, *in* *which the prices are given by*

$$P_E = \alpha_{1E} + \alpha_{2E} w_E(v_E);$$
$$P_I = \alpha_{1I} + \alpha_{2I} w_I(v_I),$$

where α_{1k} and α_{2k} are constants, and $w_k(v_k)$ is the sufficient statistic for the price information as

$$w_k(v_k) = \theta_k - \frac{a\sigma_\varepsilon^2}{v_k}(x_k - \bar{x}).$$

Furthermore, for each asset $k \in \{E, I\}$, the conditional expectation and the conditional variance of the payoff conditioning on public in formation are given by

$$E[\theta_k \mid w_k] = \bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \left(\frac{a\sigma_\varepsilon^2}{v_k}\right)\sigma_x^2}\left(w_k - \bar{\theta}\right),$$

and

$$Var[u_k \mid w_k] = \frac{\frac{a^2\sigma_\varepsilon^4}{v_k^2}\sigma_\theta^2\sigma_x^2}{\sigma_\theta^2 + \frac{a^2\sigma_\varepsilon^4}{v_k^2}\sigma_x^2} + \sigma_\varepsilon^2,$$

respectively.

See appendix A.

Theorem 1 demonstrates the existence of an equilibrium in the financial market when information processing resources are already allocated. As a direct application of Theorem 1, we can easily find that there is an equilibrium where high-type investors tilt their investments toward an efficient asset rather than an inefficient asset. Conversely, low-type investors tilt their investments toward an inefficient asset.

Let us define by β the "trading intensity" of investor i. That is, the demand of investor i can be represented as

$$X_k^i = \beta_k^i(E[u_k \mid s^i, P] - RP_k),$$

where

$$\beta_k^i = Var[u_k \mid s^i, P]^{-1}.$$

By the Bayes' rule (see appendix A for the details), the following is trivially true:

Corollary 2. *Given any allocation of information resources, the trading intensity becomes higher for the group of investors who collect more private information on that asset, i.e.,* $\beta_k^i > \beta_k^j$ *whenever* $s^i = s_k$ *and* $s^j = \emptyset$ *for all* $k \in \{E, I\}$.

This result is consistent with the intuitive notion that people tend to invest in assets they are more familiar with rather than those they are less familiar with.

2.4. Equilibrium in Attention Allocations

In previous sections, I have described the equilibrium in the financial market given capacity allocations. To characterize an overall equilibrium, I first examine the equilibrium in the financial market.

A type-τ investor's ex-ante utility before receiving signals given private information θ_E and public information P_E P_I is given by (See appendix B for the derivation.)

$$\begin{aligned} &E[E[V(W_2) \mid \theta_E, P_E, P_I]] \\ &\qquad = -\exp(aRC_E^\tau)\sqrt{\frac{Var[u_E \mid \theta_E]}{Var[u_E \mid P_E]}} \times E[\exp(-aW_2) \mid P_E, P_I] \end{aligned}$$

Likewise, a type-τ investor's ex-ante utility before receiving signals given private information θ_I and public information P_E P_I is given by

$$\begin{aligned} &E[E[V(W_2) \mid \theta_I, P_E, P_I]] \\ &\qquad = -\exp(aRC_I^\tau)\sqrt{\frac{Var[u_I \mid \theta_I]}{Var[u_I \mid P_I]}} \times E[\exp(-aW_2) \mid P_E, P_I] \end{aligned}$$

Because $Var[u_E \mid \theta_E] = Var[u_I \mid \theta_I]$, a type-$\tau$ investor would acquire information on asset E if and only if

$$\exp(aR\Delta C^{\tau}) > \sqrt{\frac{Var[u_I \mid P_I]}{Var[u_E \mid P_E]}},$$

and acquire information on asset *I* otherwise. Investors' allocation choices will include not only the cost of using resource used for interpreting signals but also the information acquired from the price, which is essentially freeriding on other investors' efforts. Intuitively, an investor would prefer the efficient asset when information processing is significantly more efficient than the inefficient asset (ΔC^{τ} is high). However, they might avoid it if the benefit is reduced due to lower precision ($Var[u_E|P_E]$ is low).

The next theorem demonstrates that one type of investors may completely specialize in one asset in equilibrium, resulting in a corner solution for investors' optimization problem in capacity allocation. This result is a direct consequence of our previous results:

Theorem 3. *In equilibrium, high-type investors specialize in the efficient asset, and low-type investors specialize in the inefficient asset if*

$$\Delta C^{L} < \frac{1}{aR}\log\left(\sqrt{\frac{Var[u_I \mid P_I]}{Var[u_E \mid P_E]}}\right) < \Delta C^{H},$$

If the price of the efficient asset becomes too informative, low-type investors may find it profitable to specialize in the inefficient asset while free-riding on high-type investors' efforts with the efficient asset. However, the theorem also demonstrates that, in any state of the world, high types cannot specialize in the inefficient asset if low types specialize in the efficient asset. This is because high types have an advantage over low types in terms of information processing (i.e., $0 < \Delta C^{L} < \Delta C^{H}$), and those with an information advantage will prioritize exploiting better opportunities first.

Using Theorem 1 and Theorem 3, we can derive the following condition for the corner equilibrium, where high types specialize in the efficient asset, and low types specialize in the inefficient asset:

$$\Delta C^{L} < \frac{\sigma_\theta^2 \upsilon_E^2 + a^2\sigma_\varepsilon^4\sigma_x^2}{aR(\sigma_\theta^2\upsilon_I^2 + a^2\sigma_\varepsilon^4\sigma_x^2)} < \Delta C^{H},$$

Intuitively, if there are too many or too few investors who have information on each asset, the equilibrium is more difficult to sustain.

When prices convey no information, investors opt for extreme allocations to the efficient asset to exploit the advantages of signal efficiency. However, if prices convey some information, low-type investors may prefer to freeride on high-type investors' efforts. This behavior is analogous to the passive investment strategy observed in the stock market, where investors focus on collecting information about a few assets while investing in some informationally efficient assets, such as a stock index, without having substantial information about it.

Since there are μ high types and $1 - \mu$ low types, the above condition provides an exact condition for the specializing equilibrium:

Theorem 4. *An overall equilibrium, where high-type investors specialize in the efficient asset, and low-type investors specialize in the inefficient asset, exists for the range of parameter values of* μ *that satisfies the following inequalities:*

$$\Delta C^{L} < \frac{\sigma_{\theta}^{2}\mu^{2} + a^{2}\sigma_{\varepsilon}^{4}\sigma_{x}^{2}}{aR(\sigma_{\theta}^{2}(1-\mu)^{2} + a^{2}\sigma_{\varepsilon}^{4}\sigma_{x}^{2})} < \Delta C^{H}.$$

The more informative the price is, the less investors will want to acquire information about that asset due to freeriding problems. If the price conveys sufficient information for freeriding, a low-type investor will focus on the inefficient asset since they can freeride on the efforts of high-type investors. If the price conveys too much information, high-type investors find less reason to stick with the efficient asset. Conversely, low-type investors find it more profitable to freeride. As the informativeness of the price for the efficient asset increases, the equilibrium where each type specializes in different assets becomes more viable. However, if the price becomes overly informative, there is no such equilibrium. Therefore, the mass of low and high investors should not be too high or too low to support this specializing equilibrium.

The theorem further highlights the importance of heterogeneity. If there are larger differences between the two groups, as captured by the difference in processing costs ΔC^{τ}, the specializing equilibrium is also more likely to exist.

Finally, I derive an equilibrium in which the heterogeneity of investors' information capacity affects their portfolio choices.

Theorem 5. *(Passive Investment Equilibrium) There exists an equilibrium where low type investors specialize in inefficient assets while high types specialize efficient assets when low type can freeride on high type's effort. As a result, low type investors tend to tilt their investments toward inefficient assets compared to high types.*

Directly derived from Corollary 2 and Theorem 4.

3. EMPIRICAL IMPLICATION OF THE RESULT

3.1. Firm Size and Signal Efficiency

In this section, we focus on stocks as a class of assets. Empirical evidence suggests that individual investors often exhibit similar portfolio choices and trading behaviors. While the direct relationship between firm size and information quality is ambiguous, the informational distinction between small and large firms may arise due to a neglect effect, as discussed by Hou and Moskowitz (2005). Research has shown that more information tends to be available for larger firms, supported by studies such as Bhushan (1989), Reburn (1994), and Zeghal (1984). This suggests that larger firms may possess a higher information-to-cost ratio, aligning with our definition of an efficient asset.

The size anomaly, where smaller firms yield higher returns after adjusting for risk, has been attributed to various factors, including liquidity, transaction costs, and information asymmetries. Hou and Moskowitz (2005) provide evidence that informational neglect may contribute to the anomaly, indicating that small stocks are often overlooked, resulting in price delays. Roll (1981) suggests that infrequent trading may lead to a mis- assessment of risk in smaller stocks, while Jagannathan and Wang (1996) propose that conditional CAPM can mitigate the size anomaly. However, a comprehensive explanation for the anomaly remains elusive. In this paper, we argue that the size anomaly may be linked to the heterogeneity of information capacity constraints. Larger stocks receive more information processing capacity, leading to greater uncertainty reduction compared to smaller stocks. Consequently, smaller stocks tend to have lower expected prices and higher returns due to unobserved risks, assuming similar fundamental factors.

Given the assumption that the fundamentals are the same, the expected price of big stock is higher than small stock if and only if the weighted average precision of big stock is higher relative to small stock. Note that both assets have the same supply variance σ_x^2, mean return, $E[u]$, and unobservable noise, σ_ε^2. In appendix C, we show that

$$E[P_E] > E[P_I];$$
$$\Leftrightarrow \frac{\mu}{\sigma_\varepsilon^2} + \frac{1-\mu}{Var[u_E \mid P_E]} > \frac{1-\mu}{\sigma_\varepsilon^2} + \frac{\mu}{Var[u_I \mid P_I]}.$$

It is observed that this condition is more likely to be satisfied as μ grows larger under the specializing equilibrium. The following corollary shows it formally.

Corollary 6. *(Size Effect) When the efficient asset is significantly more informative than the inefficient asset (when ΔC^L is sufficiently large), the size effect invariably occurs. Moreover, the size effect becomes more pronounced when μ is larger in a specializing equilibrium.*

See appendix C.

It can be interpreted that small stocks become riskier compared to large stocks as a result of capacity allocations under an equilibrium where high-type investors allocate more capacity to large stocks, and low-type investors allocate more to small stocks. Hence, small stocks require higher returns than large stocks to compensate for the unobserved risk.

In recent research, Gompers and Metrick (2001) discovered that institutional investors nearly doubled their share of the stock market from 1980 to 1996. This shift in composition has increased the demand for large firms' stocks while decreasing the demand for small firms' stocks. Consequently, this compositional shift has driven up the prices of large firms relative to small ones, aligning with the predictions of this paper. However, there's a slight difference in the interpretation of the size effect between this paper and Gompers and Metrick (2001). They interpret the price increase of large firms due to the compositional shift as the disappearance of the size effect, reasoning that the returns of large firms have risen as a result. In contrast, this paper predicts that the increased return resulting from the compositional shift is temporary and will not persist. Furthermore, this paper suggests that the risk associated with small firms has increased due to increased neglect resulting from the compositional shift, which will amplify the size effect further. The increased return of large firms due to more institutional investors is merely a transient effect that directly reflects the proportional change in investor types. Therefore, the return of large firms will return to its original level unless there is further proportional change in the future.

One prediction is that the expected price of the efficient asset (large stock) should increase as the presence of high-type investors (institutions) in the market grows. This is a natural outcome because when more institutions participate in the economy, the equilibrium results in a reduction of uncertainty for large stocks thanks to the actions of these institutions. Institutions consistently specialize in large stocks in equilibrium. As predicted in earlier sections, individual investors tend to specialize in small stocks when the price of large stocks becomes informative. The price conveys more information when the proportion of institutions is higher, as high-type investors contribute information about large stocks in equilibrium.

3.2. Initial Endowment and Home Bias

Using data from the Korean stock market, Choe, Kho, and Stulz (2005) demonstrated that foreign investors tend to execute trades at less favorable prices compared to resident investors, particularly for large trades and smaller stocks. This price disadvantage is more pronounced for sales transactions than for purchases. Additionally, their research revealed that domestic individual investors' trades contain more informational content compared to the trades of foreign investors or domestic institutional investors.

I propose that the informational advantage of domestic individual investors may be further amplified by the effect of their initial information endowment. It is possible that domestic individual investors possess specific information related to small stocks, which provides them with added motivation to specialize in these smaller stocks. As a result, domestic individual investors might possess superior information about small stocks compared to foreign institutions.

The model could be extended to offer a straightforward explanation for the familiarity effect. Suppose that each low-type investor is already granted access to a set of signals related to small stocks, denoted as θ_I, before they allocate their own information processing capacity. Furthermore, assume that this information cannot be traded. In such a scenario, low-type investors would find it more convenient to specialize in small stocks rather than large stocks. The following is a direct consequence of previous results.

Corollary 7. *(Home Bias) Given the initial endowment of signals on small stock to low type investors, the equilibrium where low type specialize in small stock becomes more feasible. That is, local investors (low type investors) are more likely to hold small stock in their local market as a result of the endowment of signals on small stock.*

This corollary illustrates how initial information endowment influences investors' capacity allocation choices. If an agent possesses information endowment related to a specific asset, it provides an additional incentive to specialize in that particular asset. This phenomenon could offer an explanation for a portion of the factors contributing to home bias in international asset markets, as well as the limited participation of some investors in the stock market (see Basak and Cuoco (1998)).

Another intriguing implication of this result pertains to cases where some investors intentionally disclose certain information publicly. The act of revealing specific information about particular assets for free is analogous to having initial endowments. Consequently, investors who receive this information may be inclined to allocate more capacity due to these initial endowments. As a result, a specific group of investors may exhibit increased demand for those assets, as demonstrated in Corollary 2. This heightened demand can drive up asset prices without any corresponding changes in their fundamental values. Therefore, if an investor already owns certain assets, they may consider selectively disclosing additional information to the public or to specific groups of investors to attract greater interest in those assets.

4. CONCLUSION

In this paper, I explore a model in which investors possess varying information processing capacities. Within this economic framework, two distinct types of assets are considered, differing in their signal efficiency. The asset with lower signal efficiency can be broadly interpreted as small stocks. In the equilibrium where prices convey no information, both types of investors specialize fully in the asset with higher signal efficiency. Conversely, in the equilibrium where prices are informative, investors with high capacity continue to specialize fully in the asset with higher signal efficiency. However, investors with lower capacity aim to capitalize on this by extracting information from the price of the high-efficiency asset while allocating their capacity resources to the asset with lower signal efficiency.

Empirical observations have consistently revealed a positive correlation between signal efficiency and firm size. This empirical evidence aligns with the predictions of this model, indicating that individual investors, with more limited information processing resources, tend to hold a larger proportion of small stocks compared to institutional investors.

Furthermore, I demonstrate that the equilibrium returns of small stocks are higher than those of large stocks due to the allocation of processing capacity. I also reveal that an individual's initial endowment of information significantly influences their equilibrium choices, effectively locking them into specializing in the asset with the initial endowment. This result can provide insights into the phenomenon of home bias in international financial markets.

These findings have broad applications, including portfolio choices among various asset classes such as stocks, bonds, and real estate. It may help explain why a substantial portion of individual investors opts for real estate investments over stocks, as real estate assets share similarities with inefficient assets where signals are less informative. Additionally, an individual's initial endowment of information about real estate could elucidate their preference for this asset class over others. This research sheds light on why some individuals abstain from participating in the stock market while others actively engage.

APPENDIX A

Proof of Theorem 1:

To prove the result, we first conjecture the given linear equilibrium prices. Under the conjecture, note that

$$E[\theta_k \mid P] = E[\theta_k \mid w_k];$$
$$Var[\theta_k \mid P] = Var[\theta_k \mid w_k],$$

for all $k \in \{E, I\}$. Therefore, it is immediate that

$$X_k(\theta_k, P_k) = \frac{\theta_k - RP_k}{a\sigma_\varepsilon^2};$$
$$X_k(\phi, P_k) = \frac{E[\theta_k \mid w_k] - RP_k}{aVar[u_k \mid w_k]}$$

Then, the market clearing conditions imply that

$$\upsilon_k \frac{\theta_k - RP_k}{a\sigma_\varepsilon^2} + (1 - \upsilon_k)\frac{E[\theta_k \mid w_k] - RP_k}{aVar[u_k \mid w_k]} = x_k.$$

Therefore, the price should satisfy

$$P_k = \frac{\upsilon_k \dfrac{\theta_k}{a\sigma_\varepsilon^2} + (1 - \upsilon_k)\dfrac{E[\theta_k \mid w_k]}{aVar[u_k \mid w_k]} - x_k}{R\left[\dfrac{\upsilon_k}{a\sigma_\varepsilon^2} + \dfrac{1 - \upsilon_k}{aVar[u_k \mid w_k]}\right]}.$$

Using the Bayes' rule under the conjectured prices, we derive the conditional expectation of the payoff:

$$E[\theta_k \mid w_k] = \bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \left(\dfrac{a\sigma_\varepsilon^2}{\upsilon_k}\right)^2 \sigma_x^2}(w_k - \bar{\theta}),$$

where

$$w_k = \theta_k - \frac{a\sigma_\varepsilon^2}{v_k}(x_k - \overline{x}).$$

Likewise, the conditional variance is

$$Var[u_k \mid w_k] = \frac{\frac{a^2\sigma_\varepsilon^4}{v_k^2}\sigma_\theta^2\sigma_x^2}{\sigma_\theta^2 + \frac{a^2\sigma_\varepsilon^4}{v_k^2}\sigma_x^2} + \sigma_\varepsilon^2$$

Then, the market clearing price can be rearranged so that

$$P_k = a_{1k} + a_{2k}w_k,$$

where a_{1k} and a_{2k} are constants. This proves the initial conjecture is indeed true.

APPENDIX B

The expected utility given s^i and P is given by

$$E[V(W_2) \mid s^i, P] = -\exp\left[\begin{array}{l} aRC_k^\tau - \frac{1}{2Var[u_E \mid s_E^i, P_E]}(E[u_E \mid s_E^i, P_E] - RP_E)^2 \\ - \frac{1}{2Var[u_I \mid s_I^i, P_I]}(E[u_I \mid s_I^i, P_I] - RP_I)^2 \end{array}\right].$$

Then, we first derive a type-τ investor's ex-ante utility when the investor receives a signal on the efficient asset as follows:

$$\begin{aligned} E[E[V(W_2) \mid \theta_E, P] \mid P] = -e^{aRC_E^\tau} & E\left[\exp\left[-\frac{1}{2Var[u_E \mid \theta_E]}(E[u_E \mid \theta_E] - RP_E)^2\right] \mid P\right] \\ & \times E\left[\exp\left[-\frac{1}{2Var[u_I \mid w_I]}(E[u_I \mid w_I] - RP_I)^2\right] \mid P\right] \\ & = -e^{aRC_E^\tau} E\left[\exp\left[-\frac{Var[\theta_E \mid P_E]}{2Var[u_E \mid \theta_E]}\frac{(E[u_E \mid \theta_E] - RP_E)^2}{Var[\theta_E \mid P_E]}\right] \mid P_E\right] \\ & \times E\left[\exp\left[-\frac{1}{2}\frac{(E[u_I \mid P_I] - RP_I)^2}{Var[u_I \mid P_I]}\right] \mid P_I\right]. \end{aligned}$$

Define

$$t \equiv \frac{Var[\theta_E \mid P_E]}{2Var[u_E \mid \theta_E]} \quad \text{and} \quad Z \equiv \frac{(E[u_E \mid \theta_E] - RP_E)^2}{Var[\theta_E \mid P_E]},$$

Then, Z has a noncentral chi-square distribution conditional on P_E. Hence, we derive the following result:

$$\begin{aligned} &E[E[V(W_2) \mid \theta_E, P] \mid P] \\ &\quad = -e^{aRC_E^\tau} E[\exp[-tZ^2] \mid P_E] E\left[\exp\left[-\frac{1}{2}\frac{(E[u_I \mid P_I] - RP_I)^2}{Var[u_I \mid P_I]}\right] \mid P_I\right] \\ &\quad = -e^{aRC_k^E} \frac{1}{\sqrt{1+2t}} \exp\left[\frac{-tE[Z_E \mid P_E]^2}{1+2t}\right] E\left[\exp\left[-\frac{1}{2}\frac{(E[u_I \mid P_I] - RP_I)^2}{Var[u_I \mid P_I]}\right] \mid P_I\right] \\ &\quad = -e^{aRC_k^E} \sqrt{\frac{Var[u_E \mid \theta_E]}{Var[u_E \mid P_E]}} \exp\left[-\left(\frac{(E[u_E \mid P_E] - RP_E)^2}{2Var[u_E \mid P_E]} + \frac{(E[u_I \mid P_I] - RP_I)^2}{2Var[u_I \mid P_I]}\right)\right]. \end{aligned}$$

Likewise, we can also derive the following ex-ante utility for the case with collecting private information on the inefficient asset:

$$\begin{aligned} &E[E[V(W_2) \mid \theta_I, P] \mid P] \\ &\quad = -e^{aRC_E^I} \sqrt{\frac{Var[u_I \mid \theta_I]}{Var[u_I \mid P_I]}} \exp\left[-\left(\frac{(E[u_E \mid P_E] - RP_E)^2}{2Var[u_E \mid P_E]} + \frac{(E[u_I \mid P_I] - RP_I)^2}{2Var[u_I \mid P_I]}\right)\right]. \end{aligned}$$

APPENDIX C

Proof of Corollary 6:

The first statement of the corollary immediately follows from the result of Theorem 3. Now, I turn to the second statement. We have the following equilibrium price given the conditional expectations and the conditional variance in case of the specializing equilibrium $\mu = \upsilon_E$:

$$P_E = \frac{\mu \dfrac{\theta_E}{a\sigma_\varepsilon^2} + (1-\mu)\dfrac{E[\theta_E \mid w_E]}{aVar[u_E \mid w_E]} - x_E}{R\left[\dfrac{\mu}{a\sigma_\varepsilon^2} + \dfrac{1-\mu}{aVar[u_E \mid w_E]}\right]}.$$

We can take unconditional expectations on both sides, and get the unconditional expectation of the price of the efficient asset:

$$E[P_E] = \frac{1}{R}\bar{\theta} - \frac{\bar{x}}{R\left[\dfrac{\mu}{a\sigma_\varepsilon^2} + \dfrac{1-\mu}{aVar[u_E \mid w_E]}\right]}.$$

Likewise, we can derive the unconditional expectation of the price of the inefficient asset:

$$E[P_I] = \frac{1}{R}\bar{\theta} - \frac{\bar{x}}{R\left[\dfrac{1-\mu}{a\sigma_\varepsilon^2} + \dfrac{\mu}{aVar[u_I \mid w_I]}\right]}.$$

The first term represents the present value of the expected cash flow, and the second term represents the discount due to uncertainty.

It is immediate that the second term in $E[P_E]$decreases (i.e., $E[P_E]$ increases) under two conditions: (i) when μ increases while keeping $Var[u_E | w_E]$ fixed, due to $\sigma_\varepsilon^2 < Var[u_E | w_E]$, or (ii) when the uncertainty conditioned on the price ($Var[u_E | w_E]$) further decreases. Additionally, $Var[u_E | w_E]$ diminishes as μ increases, following from Theorem 1. Consequently, we deduce that $E[P_E]$ increases with μ. A similar rationale applies to the second term in $E[P_I]$ concerning $1 - \mu$, indicating that $E[P_I]$ decreases with μ. Thus, these conclusions complete the proofs.

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